Taylor3-3c.tex, 160123.

$f(x,y,z)$ 3-rendű Taylor-polinomja

$$f:\mathbb{R}^3 \to \mathbb{R}\ , \quad \boxed{f(x,y,z) = \frac{x^2}{z}\cdot \exp\left(y^3\right)\ , \quad \underline{a} = (2,1,5)\ ,}$$

$$f(x,y,z) = \frac{x^2}{z}\cdot \exp\left(y^3\right) \qquad \Rightarrow \qquad f\left(\underline{a}\right) = \frac{x^2}{z}\cdot \exp\left(y^3\right) = \frac{2^2}{5}e^{1^3} \approx \mathbf{2.1746},$$

$$D_x f = \frac{d}{dx}\left(\frac{x^2}{z}\cdot \exp\left(y^3\right)\right) = 2\frac{x}{z}e^{y^3} \Rightarrow \left(D_x f\right)\left(\underline{a}\right) = 2\cdot\frac{2}{5}e^{1^3} \approx \mathbf{2.1746}\ \text{(ok)},$$

$$D_y f = \frac{d}{dy}\left(\frac{x^2}{z}\cdot \exp\left(y^3\right)\right) = \frac{3x^2y^2}{z}e^{y^3} \Rightarrow \left(D_y f\right)\left(\underline{a}\right) = \frac{3\cdot 2^2\cdot 1^2}{5}e^{1^3} \approx \mathbf{6.5239}$$

$$D_z f = \frac{d}{dz}\left(\frac{x^2}{z}\cdot \exp\left(y^3\right)\right) = \frac{-x^2}{z^2}e^{y^3} \quad \Rightarrow \quad \left(D_z f\right)\left(\underline{a}\right) = \frac{-2^2}{5^2}e^{1^3} \approx \mathbf{-0.4349},$$

$$\left(D^2_{xx}\right)f = \frac{d^2}{dx^2}\left(\frac{x^2}{z}\cdot \exp\left(y^3\right)\right) = \frac{2}{z}e^{y^3} \quad \Rightarrow \quad \left(D^2_{xx} f\right)\left(\underline{a}\right) = \frac{2}{5}e^{1^3} \approx \mathbf{1.0873}$$

$$\left(D^2_{yy}\right)f = \frac{d^2}{dy^2}\left(\frac{x^2}{z}\cdot \exp\left(y^3\right)\right) = 3x^2\frac{y}{z}e^{y^3}\left(3y^3+2\right) \quad \Rightarrow$$

$$\Rightarrow \quad \left(D^2_{yy} f\right)\left(\underline{a}\right) = 3\cdot 2^2\cdot\frac{1}{5}e^{1^3}\left(3\cdot 1^3+2\right) \approx \mathbf{32.619}$$

$$\left(D^2_{zz}\right)f = \frac{d^2}{dz^2}\left(\frac{x^2}{z}\cdot \exp\left(y^3\right)\right) = 2\frac{x^2}{z^3}e^{y^3} \Rightarrow \left(D^2_{zz} f\right)\left(\underline{a}\right) = 2\cdot\frac{2^2}{5^3}e^{1^3} \approx \mathbf{0.1739}$$

$$\left(D^2_{xy}\right)f = \frac{d^2}{dxy}\left(\frac{x^2}{z}\cdot \exp\left(y^3\right)\right) = 6x\frac{y^2}{z}e^{y^3} \Rightarrow \left(D^2_{xy} f\right)\left(\underline{a}\right) = 6\cdot 2\frac{1^2}{5}e^{1^3} \approx \mathbf{6.5239}$$

$$\left(D^2_{xz}\right)f = \frac{d^2}{dxz}\left(\frac{x^2}{z}\cdot \exp\left(y^3\right)\right) = -2\frac{x}{z^2}e^{y^3} \Rightarrow \left(D^2_{xz} f\right)\left(\underline{a}\right) = -2\frac{2}{1^2}e^{1^3} \approx \mathbf{-10.873},$$

$$\left(D^2_{yz}\right)f = \frac{d^2}{dyz}\left(\frac{x^2}{z}\cdot \exp\left(y^3\right)\right) = -3x^2\frac{y^2}{z^2}e^{y^3} \Rightarrow \left(D^2_{yz} f\right)\left(\underline{a}\right) = -3\cdot 2^2\frac{1^2}{5^2}e^{1^3} \approx \mathbf{-1.3048},$$

$$\left(D^3_{xxx}\right)f = \frac{d^3}{dx^3}\left(\frac{x^2}{z}\cdot \exp\left(y^3\right)\right) = 0 \quad \Rightarrow \quad \left(D^3_{xxx} f\right)\left(\underline{a}\right) = \mathbf{0}\ ,$$

$$\left(D^3_{yyy}\right)f = \frac{d^3}{dy^3}\left(\frac{x^2}{z}\cdot \exp\left(y^3\right)\right) = 3\frac{x^2}{z}e^{y^3}\left(9y^6+18y^3+2\right) \quad \Rightarrow$$

$$\Rightarrow \quad \left(D^3_{yyy} f\right)\left(\underline{a}\right) = 3\cdot\frac{2^2}{5}e^{1^3}\left(9\cdot 1^6+18\cdot 1^3+2\right) \approx \mathbf{189.1924},$$

$$\left(D^3_{zzz}\right)f = \frac{d^3}{dz^3}\left(\frac{x^2}{z}\cdot \exp\left(y^3\right)\right) = -6\frac{x^2}{z^4}e^{y^3} \Rightarrow \left(D^3_{zzz} f\right)\left(\underline{a}\right) = -6\cdot\frac{2^2}{5^4}e^{1^3} \approx \mathbf{-0.1044},$$

$$\left(D^3_{xxy}\right)f = \frac{d^3}{dx^2y}\left(\frac{x^2}{z}\cdot \exp\left(y^3\right)\right) = 6\frac{y^2}{z}e^{y^3} \Rightarrow \left(D^3_{xxy} f\right)\left(\underline{a}\right) = 6\cdot\frac{1^2}{5}e^{1^3} \approx \mathbf{3.2619},$$

$$\left(D^3_{xyy}\right)f = \frac{d^3}{dxy^2}\left(\frac{x^2}{z}\cdot \exp\left(y^3\right)\right) = \frac{1}{z}\left(12xye^{y^3}+18xy^4e^{y^3}\right) \quad \Rightarrow$$

$$\Rightarrow \quad \left(D^3_{xyy} f\right)\left(\underline{a}\right) = \frac{1}{5}\cdot\left(12\cdot 2e^{1^3}+18\cdot 2\cdot 1^4e^{1^3}\right) \approx \mathbf{32.6193},$$

$$\left(D^3_{xxz}\right) f = \frac{d^3}{dx^2 z}\left(\frac{x^2}{z}\cdot \exp\left(y^3\right)\right) = -\frac{2}{z^2}e^{y^3} \Rightarrow \left(D^3_{xxz} f\right)(\underline{a}) = -\frac{2}{5^2}e^{1^3} \approx \mathbf{-0.2175},$$

$$\left(D^3_{xzz}\right) f = \frac{d^3}{dxz^2}\left(\frac{x^2}{z}\cdot \exp\left(y^3\right)\right) = 4\frac{x}{z^3}e^{y^3} \Rightarrow \left(D^3_{xzz} f\right)(\underline{a}) = 4\cdot \frac{2}{5^3}e^{1^3} \approx \mathbf{0.1739},$$

$$\left(D^3_{yyz}\right) f = \frac{d^3}{dy^2 z}\left(\frac{x^2}{z}\cdot \exp\left(y^3\right)\right) = -3x^2\frac{y}{z^2}e^{y^3}\left(3y^3+2\right) \quad \Rightarrow$$

$$\Rightarrow \quad \left(D^3_{yyz} f\right)(\underline{a}) = -3\cdot 2^2\frac{1}{5^2}e^{1^3}\left(3\cdot 1^3+2\right) \approx \mathbf{-6.5239}$$

$$\left(D^3_{yzz}\right) f = \frac{d^3}{dyz^2}\left(\frac{x^2}{z}\cdot \exp\left(y^3\right)\right) = 6x^2\frac{y^2}{z^3}e^{y^3} \Rightarrow \left(D^3_{yzz} f\right)(\underline{a}) = 6\cdot 2^2\frac{1^2}{5^3}e^{1^3} \approx \mathbf{0.5219}$$

$$\left(D^3_{xyz}\right) f = \frac{d^3}{dxyz}\left(\frac{x^2}{z}\cdot \exp\left(y^3\right)\right) = -6x\frac{y^2}{z^2}e^{y^3} \quad \Rightarrow$$

$$\Rightarrow \left(D^3_{xyz} f\right)(\underline{a}) = -6\cdot 2\cdot \frac{1^2}{5^2}e^{1^3} \approx \mathbf{-1.3048}$$

Tehát :

$$\left(T^3_{\underline{a}}, f\right)(x,y,z) \approx 2.1746 + 2.1746(\mathbf{x-2}) + 6.5239(\mathbf{y+1}) - 0.4349(\mathbf{z-5}) +$$

$$+ \frac{1}{2!}\left(1.0873(\mathbf{x-2})^2 + 32.619(\mathbf{y+1})^2 + 0.1739(\mathbf{z-5})^2\right) +$$

$$+ \frac{2}{2!}\left(6.5239(\mathbf{x-2})^1(\mathbf{y+1})^1 - 10.873(\mathbf{x-2})^1(\mathbf{z-5})^1 - 1.3048(\mathbf{y+1})^1(\mathbf{z-5})^1\right) +$$

$$+ \frac{1}{3!}\left(0\cdot(\mathbf{x-2})^3 + 189.1924\,(\mathbf{y+1})^3 - 0.1044\,(\mathbf{z-5})^3\right) +$$

$$+ \frac{3}{3!}\left(3.2619(\mathbf{x-2})^2(\mathbf{y+1}) + 32.6193(\mathbf{x-2})(\mathbf{y+1})^2 - 0.2175(\mathbf{x-2})^2(\mathbf{z-5})\right) +$$

$$+ \frac{3}{3!}\left(0.1739(\mathbf{x-2})(\mathbf{z-5})^2 - 6.5239(\mathbf{y+1})^2(\mathbf{z-5}) + 0.5219(\mathbf{y+1})(\mathbf{z-5})^2\right) +$$

$$+ \frac{6}{3!}\left(-1.3048(\mathbf{x-2})^1(\mathbf{y+1})^1(\mathbf{z-5})^1\right)$$