$f(x,y,z)$ 3-rendű Taylor-polinomja , (2008.03.18.)

$f:\mathbb{R}^3 \to \mathbb{R}$, $f(x,y,z) = \frac{x^2y}{x+3z}$, $a = (2,-1,8)$,

$$f(x,y,z) = x^2y/(x+3z) \Rightarrow f(a) = -4/26 ,$$

$$(D_x)f = y - 9yz^2/(x+3z)^2 \Rightarrow [(D_x)f](a) = -25/169,$$

$$(D_y)f = 9z^2/(x+3z) + x - 3z \Rightarrow [(D_y)f](a) = 2/13 ,$$

$$(D_z)f = -27yz^2/(x+3z)^2 + 18yz/(x+3z) - 3y \Rightarrow [(D_z)f](a) = 3/169,$$

$$(D^2_{xx})f = 18yz^2/(x{+}3z)^3 \Rightarrow [(D^2_{xx})f](a) = -144/2197,$$

$$(D^2_{yy})f = 0 \Rightarrow [(D^2_{yy})f](a) = 0 ,$$

$$(D^2_{zz})f = 162yz^2/(x+3z)^3 - 108yz/(x+3z)^2 + 18y/(x+3z)$$
$$\Rightarrow [(D^2_{zz})f](a) = -9/2197,$$

$$(D_{xy})f = 1 - 9z^2/(x+3z)^2 \Rightarrow [(D_{xy})f](a) = 25/169,$$

$$(D_{xz})f = 54yz^2/(x+3z)^3 - 18yz/(x+3z)^2 \Rightarrow [(D_{xz})f](a) = 36/2197,$$

$$(D_{yz})f = -27z^2/(x+3z)^2 + 18z/(x+3z) - 3 \Rightarrow [(D_{yz})f](a) = -3/169,$$

$$(D^3_{xxx})f = -54yz^2/(x{+}3z)^4 \Rightarrow [(D^3_{xxx})f](a) = 216/28561,$$

$$(D^3_{yyy})f = 0 \Rightarrow [(D^3_{yyy})f](a) = 0 ,$$

$$(D^3_{zzz})f = -1458yz^2/(x+3z)^4 + 972yz/(x+3z)^3 - 162y/(x+3z)^2$$
$$\Rightarrow [(D^3_{zzz})f](a) = 81/57122,$$

$$(D^2_{xx}D_y)f = 18z^2/(x{+}3z)^3 \Rightarrow [(D^2_{xx}D_y)f](a) = 144/2197,$$

$$(D^2_{yy}D_x)f = 0 \Rightarrow [(D^2_{yy}D_x)f](a) = 0 ,$$

$$(D^2_{xx}D_z)f = 36yz/(x{+}3z)^3 {-} 162yz^2/(x{+}3z)^4 \Rightarrow [(D^2_{xx}D_z)f](a) = 180/28561,$$

$$(D^2_{zz}D_x)f = -486yz^2/(x+3z)^4 + 216yz/(x+3z)^3 - 18y/(x+3z)^2$$
$$\Rightarrow [(D^2_{zz}D_x)f](a) = -207/57122,$$

$$(D^2_{yy}D_z)f = 0 \Rightarrow [(D^2_{yy}D_z)f](a) = 0 ,$$

$$(D^2_{zz}D_y)f = 162z^2/(x+3z)^3 - 108z/(x+3z)^2 + 18/(x+3z)$$
$$\Rightarrow [(D^2_{zz}D_y)f](a) = \tfrac{9}{2197},$$

$$(D_xD_yD_z)f = 54z^2/(x{+}3z)^3 {-} 18z/(x{+}3z)^2 \Rightarrow [(D_xD_zD_y)f](a) = \tfrac{-36}{2197},$$

tehát $\quad (T_a^3, f)\,(x,y,z) =$

$$\frac{-4/26}{0!}\cdot 1 + \frac{-25/169}{1!}(x-2) + \frac{2/13}{1!}(y+1) + \frac{3/169}{1!}(z-8) +$$
$$+\frac{-144/2197}{2!}(x-2)^2 + \frac{0}{2!}(y+1)^2 + \frac{-9/2197}{2!}(z-8)^2 +$$
$$+\frac{25/169}{2!}(x-2)(y+1) + \frac{36/2197}{2!}(x-2)(y+1) + \frac{-3/169}{2!}(y+1)(z-8) +$$
$$+\frac{216/28561}{3!}(x-2)^3 + \frac{0}{3!}(y+1)^3 + \frac{81/57122}{3!}(z-8)^3 +$$
$$+\frac{144/2197}{3!}(x-2)^2(y+1) + \frac{0}{3!}(y+1)^2(x-2) + \frac{180/28561}{3!}(x-2)^2(z-8) +$$
$$+\frac{-207/57122}{3!}(z-8)^2(x-2) + \frac{0}{3!}(y+1)^2(z-8) + \frac{9/2197}{3!}(y+1)^2(z-8) +$$
$$+\frac{-36/2197}{3!}(x-2)(y+1)(z-8) \approx$$

$\approx$

$$-0.1538 - 0.\,1479(x-2) + 0.153\,8(y+1) + 0.0178(z-8) +$$
$$-0.0328(x-2)^2 - 0.0020(z-8)^2 + 0.0739(x-2)(y+1) +$$
$$+0.0082(x-2)(y+1) - 0.0089(y+1)(z-8) + 0.0013(x-2)^3 +$$
$$+0.0002(z-8)^3 + 0.0109(x-2)^2(y+1) + 0.0011(x-2)^2(z-8) +$$
$$-0.0006(z-8)^2(x-2) + 0.0007(y+1)^2(z-8) - 0.0027(x-2)(y+1)(z-8) \quad .$$