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Higher Gap Simplified Morasses and Combinatorial Applications

(PhD-Thesis, 1991, in Hungarian)

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Extended abstract

In this PhD-Thesis we introduce the notion of the **gap-m** ($m < \omega$) **simplified morasses**, investigate their properties, and prove some theorems, illustrating their applications in combinatorial set theory.

Chapter 1 *The definitions of gap-1 and higher-gap, ordinary and simplified morasses.*
After running through the definitions of gap-1, gap-2 and higher-gap, both ordinary and simplified morasses (using the results of R.B.Jensen [Je1],[Je2] and D.J.Velleman [Ve2],[Ve8]), we introduce the notion of the general gap-m **simplified morasses** $\mathcal{M}$ of height κ , shortly (κ, m)**-morasses** for any $m < \omega_0$ and κ regular cardinal (Definitions 1.26-1.32).

Chapter 2 *Properties of gap-1 and higher-gap simplified morasses.*
In this chapter we prove several elementary properties of the simplified morasses. We also present some short, direct proof for the statements concerning small gap morasses (eg. Statement 2.6), or *construct a super Souslin tree* using a simplified morass (Statement 2.6). We provide some new statements, eg.:

Statement 2.7 *If* $\mathcal{M} =< \overset{\Rightarrow}{\mathcal{F}_{\alpha,\beta}}, \vec{\varphi} >$ *is a gap-1 simplified morass, then* $\varphi_\beta > sup\{\varphi_\alpha : \alpha < \beta\}$ *for some limit* α. □

We also present the unpublished result of prof.R.B.Jensen:

Statement 2.8 (Jensen [Je2]): *There exists* no *simplified* $(\omega_1, 1)$ *morass* $\mathcal{M} =< \overset{\Rightarrow}{\mathcal{F}_{\alpha,\beta}}, \vec{\varphi} >$ *in which the sequence* $\langle \sigma_\alpha : \alpha < ht(\mathcal{M}) \rangle$ *of the splitting points would be increasing.* □

The properties of the higher gap simplified morasses are described in Statements 2.20 through 2.41. We use these results in the last two chapters.

Chapter 3 *The free subset problem*
The chapter is devoted to the proof of the following result:

Theorem 3.2 *Supposing* $m < \omega_0$, CH *and the existence an* (ω_1, m) *-morass with full linearizing sequences (Definition 2.42), then* not-Free$(\omega_{m+1}, 2^m + 1, \omega_1)$ *holds* (Definition 3.0) □

This result answers (in a week form) a conjecture of Hajnal and Komjáth [HK].

Chapter 4 *A partition relation*
In this chapter we prove:

Theorem 4.8 *If* $2^{\aleph_1} = \aleph_2$ *and there exists an* $(\omega_2, 1)$ *-morass with linearizing sequences, then* $\omega_3\omega_1 \not\to (\omega_3\omega_1, 3)^2$. □

This answers the conjecture of S.Shelah and L.J.Stanley [SS2]. We made use of the following result:

Statement 4.12 *μ is finite, where $\mu = \mu(f, g)$ is the number of the levels where two functions $f, g \in \mathcal{F}$ in a simplified morass are different* (Definition 4.11). $\square$

Chapter 5 *The existence of higher gap simplified morasses.*
In this chapter we derive the existence of higher gap morasses from the existence of (ordinary) higher gap morasses, using one of Ch.Morgan's results.

Major parts ot this Thesis are appeared in [Sz2].

Content of the Thesis

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