Deriváló

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Derivalo-mego.tex, 2018.11.01.

http://math.uni-pannon.hu/~szalkai/Derivalo-mego-181101.pdf

Számoljuk ki az alábbi függvények deriváltfüggvényeit ("formális deriválás")! Ügyeljünk a zárójelekre!

1.) $(x^2+3x-1)\left(x+\sqrt{2}\right)$, $(x-1)^2\,(x^2+2)$,

2.) $\dfrac{3x^2-2x+1}{\sqrt[3]{x}}$, $\dfrac{(x+1)^2}{x^2+1}$, $\dfrac{\sqrt{x}-x^3}{\sqrt[3]{x}}$,

3. !) $\left(\dfrac{x+1}{x-1}\right)^2$,

4. !!!) $\dfrac{2}{3+8x^5}$, $\dfrac{1}{\tan^2 x}$,

5. !) $\dfrac{1}{\sqrt{x}}-3\sqrt{x^2+5x}$, $\sqrt[3]{1+\sqrt[3]{2+5x}}$,

6.) $(3x^2+5x)\cos 3x$, $(2+x^2)\sin 2x$,

7. !!!) $e^{\sqrt{1+x}}$, 5^{1-x^2} , $3^{1/x}$,

8.) $\dfrac{1+\cos x}{1-\cos x}$, $\dfrac{\cos^2 x}{\cos x^2}$,

9.) $\sqrt{\sin x}$, $\sin\sqrt{x}$, $\sqrt[7]{chx}$,

10.) $\arcsin\dfrac{1}{x}$, $\arccos\left(1-x^2\right)$, $\arcsin\left(2x\sqrt{1-x^2}\right)$,

11. !!) $\ln\dfrac{x}{2+3x}$, $\ln\tan x$,

12. !!!) $\ln\sqrt{\dfrac{5x}{2+3x}}$, $\log_2\sqrt{\dfrac{5x}{2+3x}}$,

13.) $\dfrac{x}{\sin x + \cos x}$,

14. !!!) $(3x^2 - 2)\, e^2$,

15.) $(3x^2 - 2)\, e^{2x}$,

16.) $e^{2x} \cos 3x$,

17. !!!) $\tan \dfrac{1}{x^2}$, $x^2 \tan 2x$,

18. *) $\dfrac{1}{chx}$,

19.) $\log_3 (1 - x^2)$,

20.) $\ln \dfrac{x^2 - 1}{x^2 + 1}$,

21.) $\sqrt{\dfrac{1 + \cos x}{2}}$,

22.) $x^3 (1 - 2x)(3x^2 + 4x)$,

23.) ÚJ: $\dfrac{\sin(\cos(x^2))}{\exp_4\left(tg\left(\dfrac{1}{x^2}\right)\right)}$.

Megoldások

1)a) $\left[(x^2+3x-1)\left(x+\sqrt{2}\right)\right]'=(x^2+3x-1)'\left(x+\sqrt{2}\right)+(x^2+3x-1)\left(x+\sqrt{2}\right)'=$
$=(2x+3)\left(x+\sqrt{2}\right)+(x^2+3x-1)\cdot 1$,

b) $\left[(x-1)^2(x^2+2)\right]'=\left[(x-1)^2\right]'(x^2+2)+(x-1)^2(x^2+2)'=$
$=2\cdot(x-1)^{2-1}\cdot 1+(x-1)^2(2x+0)$

2)a) $\left(\dfrac{3x^2-2x+1}{\sqrt[3]{x}}\right)'=\dfrac{(3x^2-2x+1)'\cdot\sqrt[3]{x}-(3x^2-2x+1)\cdot\left(x^{1/3}\right)'}{\left(\sqrt[3]{x}\right)^2}=$

$$=\frac{(3\cdot 2\cdot x^{2-1}-2+0)\cdot\sqrt[3]{x}-(3x^2-2x+1)\cdot\left(\frac{1}{3}x^{1/3-1}\right)}{\left(\sqrt[3]{x}\right)^2}$$

b) $\left(\dfrac{(x+1)^2}{x^2+1}\right)'=\dfrac{\left[(x+1)^2\right]'\cdot(x^2+1)-(x+1)^2\cdot(x^2+1)'}{(x^2+1)^2}=$

$$=\frac{\left[2\cdot(x+1)^{2-1}\cdot 1\right]\cdot(x^2+1)-(x+1)^2\cdot(2x^{2-1}+0)}{(x^2+1)^2}\ ,$$

c) $\left(\dfrac{\sqrt{x}-x^3}{\sqrt[3]{x}}\right)'=\dfrac{\left(x^{1/2}-x^3\right)'\cdot\sqrt[3]{x}-(\sqrt{x}-x^3)\cdot\left(x^{1/3}\right)'}{\left(\sqrt[3]{x}\right)^2}=$

$$=\frac{\left(\frac{1}{2}x^{\frac{1}{2}-1}-3x^{3-1}\right)\cdot\sqrt[3]{x}-(\sqrt{x}-x^3)\cdot\left(\frac{1}{3}x^{1/3-1}\right)}{\left(\sqrt[3]{x}\right)^2}\ ,$$

3) $\left[\left(\dfrac{x+1}{x-1}\right)^2\right]'=2\cdot\left(\dfrac{x+1}{x-1}\right)^{2-1}\cdot\left(\dfrac{x+1}{x-1}\right)'=$

$$=2\cdot\left(\frac{x+1}{x-1}\right)^{2-1}\cdot\frac{(x+1)'\cdot(x-1)-(x+1)\cdot(x-1)'}{(x-1)^2}=2\cdot\left(\frac{x+1}{x-1}\right)^{2-1}\cdot\frac{1\cdot(x-1)-(x+1)\cdot 1}{(x-1)^2}$$

4)a) $\left(\dfrac{2}{3+8x^5}\right)'=\left(2\cdot(3+8x^5)^{-1}\right)'=2\cdot(-1)\cdot(3+8x^5)^{-1-1}\cdot(0+8\cdot 5x^{5-1})$,

b) $\left(\dfrac{1}{\tan^2 x}\right)'=\left((\tan^2(x))^{-1}\right)'=\left((\tan(x))^{-2}\right)'=-2\cdot(\tan(x))^{-2-1}\cdot\tan'(x)=$

$$=-2\cdot(\tan(x))^{-2-1}\cdot\frac{1}{\cos^2(x)}\ ,$$

5)a) $\left(\dfrac{1}{\sqrt{x}}-3\sqrt{x^2+5x}\right)'=\left(x^{-\frac{1}{2}}\right)'-3\left((x^2+5x)^{\frac{1}{2}}\right)'=$

$$=\frac{-1}{2}\cdot x^{-\frac{1}{2}}-3\cdot\frac{1}{2}\cdot(x^2+5x)^{\frac{1}{2}-1}\cdot(2x+5)$$

b) $\left(\sqrt[3]{1+\sqrt[3]{2+5x}}\right)'=\left(\left(1+(2+5x)^{\frac{1}{3}}\right)^{\frac{1}{3}}\right)'=$

$$=\frac{1}{3}\cdot\left(1+(2+5x)^{\frac{1}{3}}\right)^{\frac{1}{3}-1}\cdot\left(0+\frac{1}{3}\cdot(2+5x)^{\frac{1}{3}-1}\cdot(0+5)\right)\ ,$$

6)a) $[(3x^2+5x)\cdot\cos(3x)]' = (3x^2+5x)'\cdot\cos(3x) + (3x^2+5x)\cdot(\cos(3x))' =$
$= (3\cdot 2x^{2-1}+5)\cdot\cos(3x) + (3x^2+5x)\cdot(-\sin(3x))\cdot 3$,

b) $[(2+x^2)\cdot\sin(2x)]' = (2+x^2)'\cdot\sin(2x) + (2+x^2)\cdot(\sin(2x))' =$
$= (0+2x^{2-1})\cdot\sin(2x) + (2+x^2)\cdot\cos(2x)\cdot 2$,

7)a) $\left(e^{\sqrt{1+x}}\right)' = \left(\exp\left(\sqrt{1+x}\right)\right)' = \exp'\left(\sqrt{1+x}\right)\cdot\left(\sqrt{1+x}\right)' = e^{\sqrt{1+x}}\cdot\left((1+x)^{\frac{1}{2}}\right)' =$
$= e^{\sqrt{1+x}}\cdot\frac{1}{2}(1+x)^{\frac{1}{2}-1}\cdot 1$

b) $\left(5^{1-x^2}\right)' = (\exp_5(1-x^2))' = \exp_5'(1-x^2)\cdot(1-x^2)' = 5^{1-x^2}\cdot\ln(5)\cdot(1-2x)$,

c) $\left(3^{1/x}\right)' = \left(\exp_3\left(\frac{1}{x}\right)\right)' = \exp_3'\left(\frac{1}{x}\right)\cdot(x^{-1})' = 3^{1/x}\cdot\ln(3)\cdot(-1)\cdot 2\cdot x^{-1-1}$

8)a) $\left(\frac{1+\cos x}{1-\cos x}\right)' = \frac{(1+\cos x)'\cdot(1-\cos x)-(1+\cos x)\cdot(1-\cos x)'}{(1-\cos x)^2} =$
$= \frac{(0-\sin x)\cdot(1-\cos x)-(1+\cos x)\cdot(0+\sin x)}{(1-\cos x)^2}$,

b) $\left(\frac{\cos^2 x}{\cos x^2}\right)' = \frac{\left([\cos(x)]^2\right)'\cdot(\cos(x^2)) - \left([\cos(x)]^2\right)\cdot(\cos(x^2))'}{(\cos(x^2))^2} =$
$= \frac{2\cdot[\cos(x)]^{2-1}(-\sin x)\cdot(\cos(x^2)) - \left([\cos(x)]^2\right)\cdot(-\sin(x^2)\cdot 2x)}{(\cos(x^2))^2}$,

9)a) $\left(\sqrt{\sin x}\right)' = \left((\sin(x))^{\frac{1}{2}}\right)' = \frac{1}{2}\cdot(\sin(x))^{\frac{1}{2}-1}\cdot\sin'(x) = \frac{1}{2}\cdot(\sin(x))^{\frac{1}{2}-1}\cdot\cos(x)$

b) $\left(\sin\left(x^{\frac{1}{2}}\right)\right)' = \cos\left(x^{\frac{1}{2}}\right)\cdot\frac{1}{2}\cdot x^{\frac{1}{2}-1}$

c) $\left(\sqrt[7]{chx}\right)' = \left((ch(x))^{\frac{1}{7}}\right)' = \frac{1}{7}\cdot(ch(x))^{\frac{1}{7}-1}\cdot sh(x)$

10)a) $\left(\arcsin\frac{1}{x}\right)' = \arcsin'\left(\frac{1}{x}\right)\cdot\left(\frac{1}{x}\right)' = \frac{1}{\sqrt{1-\left(\frac{1}{x}\right)^2}}\cdot(-1)\cdot x^{-1-1}$,

b) $(\arccos(1-x^2))' = \frac{-1}{\sqrt{1-(1-x^2)^2}}\cdot(0-2x)$,

c) $\left(\arcsin\left(2x\cdot\sqrt{1-x^2}\right)\right)' = \arcsin'\left(2x\cdot\sqrt{1-x^2}\right)\cdot\left(2x\cdot\sqrt{1-x^2}\right)' =$
$= \frac{1}{\sqrt{1-\left(2x\sqrt{1-x^2}\right)^2}}\cdot\left((2x)'\cdot\sqrt{1-x^2}+2x\cdot\left((1-x^2)^{\frac{1}{2}}\right)'\right) =$
$= \frac{1}{\sqrt{1-\left(2x\sqrt{1-x^2}\right)^2}}\cdot\left(2\cdot\sqrt{1-x^2}+2x\cdot\frac{1}{2}\cdot(1-x^2)^{\frac{1}{2}-1}\cdot(0-2x)\right)$,

11)a) $\left(\ln\frac{x}{2+3x}\right)' = \ln'\left(\frac{x}{2+3x}\right)\cdot\left(\frac{x}{2+3x}\right)' = \frac{1}{\frac{x}{2+3x}}\cdot\frac{x'\cdot(2+3x)-x\cdot(2+3x)'}{(2+3x)^2} =$

$= \frac{2+3x}{x}\cdot\frac{1\cdot(2+3x)-x\cdot 3}{(2+3x)^2}$,

b) $[\ln(\tan(x))]' = \ln'(\tan(x))\cdot\tan'(x) = \frac{1}{\tan(x)}\cdot\frac{1}{\cos^2(x)}$,

12)a) $\left(\ln\sqrt{\frac{5x}{2+3x}}\right)' = \ln'\left(\sqrt{\frac{5x}{2+3x}}\right)\cdot\left(\sqrt{\ }'\left(\frac{5x}{2+3x}\right)\right)\cdot\left(\frac{5x}{2+3x}\right)' =$

$= \frac{1}{\sqrt{\frac{5x}{2+3x}}}\cdot\frac{1}{2\sqrt{\frac{5x}{2+3x}}}\cdot\frac{(5x)'\cdot(2+3x)-5x\cdot(2+3x)'}{(2+3x)^2} =$

$= \frac{1}{\sqrt{\frac{5x}{2+3x}}}\cdot\frac{1}{2\sqrt{\frac{5x}{2+3x}}}\cdot\frac{5\cdot(2+3x)-5x\cdot 3x}{(2+3x)^2}$,

b) $\left(\log_2\sqrt{\frac{5x}{2+3x}}\right)' = \frac{1}{\sqrt{\frac{5x}{2+3x}}\cdot\ln(2)}\cdot\frac{1}{2\sqrt{\frac{5x}{2+3x}}}\cdot\frac{5\cdot(2+3x)-5x\cdot 3x}{(2+3x)^2}$,

13) $\left(\frac{x}{\sin x+\cos x}\right)' = \frac{x'\cdot(\sin x+\cos x)-x\cdot(\sin x+\cos x)'}{(\sin x+\cos x)^2} =$

$= \frac{1\cdot(\sin x+\cos x)-x\cdot(\cos x-\sin x)}{(\sin x+\cos x)^2}$,

14) $[(3x^2-2)\cdot e^2]' = (3x^2-2)'\cdot e^2 = (3\cdot 2\cdot x-0)\cdot e^2 = 6x\cdot e^2$,

15) $[(3x^2-2)\cdot e^{2x}]' = (3x^2-2)'\cdot e^{2x}+(3x^2-2)\cdot(e^{2x})' = (3\cdot 2\cdot x-0)\cdot e^{2x}+(3x^2-2)\cdot e^{2x}\cdot 2$

16) $[e^{2x}\cdot\cos 3x]' = (e^{2x})'\cdot\cos 3x+e^{2x}\cdot(\cos 3x)' = e^{2x}\cdot 2\cdot\cos 3x+e^{2x}\cdot(-\sin(3x))\cdot 3$,

17)a) $\left(\tan\left(\frac{1}{x^2}\right)\right)' = \tan'\left(\frac{1}{x^2}\right)\cdot\left(\frac{1}{x^2}\right)' = \frac{1}{\cos^2\left(\frac{1}{x^2}\right)}\cdot\frac{-2}{x^3}$,

b) $[x^2\cdot\tan(2x)]' = (x^2)'\cdot\tan(2x)+x^2\cdot\tan'(2x)\cdot(2x)' = 2x\cdot\tan(2x)+x^2\cdot\frac{1}{\cos^2(2x)}\cdot 2$,

18) $\left(\frac{1}{ch(x)}\right)' = \left(\frac{2}{e^x+e^{-x}}\right)' = 2\cdot\left[(e^x+e^{-x})^{-1}\right]' = 2\cdot(-1)\cdot(e^x+e^{-x})^{-2} = \frac{-2}{(e^x+e^{-x})^2}$,

19) $[\log_3(1-x^2)]' = \log_3'(1-x^2)\cdot(1-x^2)' = \frac{1}{(1-x^2)\cdot\ln(3)}\cdot(0-2x)$,

20) $\left(\ln\left(\frac{x^2-1}{x^2+1}\right)\right)' = \ln'\left(\frac{x^2-1}{x^2+1}\right)\cdot\left(\frac{x^2-1}{x^2+1}\right)' = \frac{x^2+1}{x^2-1}\cdot\frac{2x\cdot(x^2+1)-2x\cdot(x^2-1)}{(x^2+1)^2}$,

21) $\left(\sqrt{\frac{1+\cos x}{2}}\right)' = \sqrt{\ }'\left(\frac{1+\cos x}{2}\right)\cdot\left(\frac{1+\cos x}{2}\right)' = \frac{1}{2\sqrt{\frac{1+\cos x}{2}}}\cdot\frac{1}{2}\cdot(0-\sin(x))$,

22) $[x^3 \cdot (1-2x) \cdot (3x^2+4x)]' =$

$= (x^3)' \cdot (1-2x) \cdot (3x^2+4x) + x^3 \cdot (1-2x)' \cdot (3x^2+4x) + x^3 \cdot (1-2x) \cdot (3x^2+4x)' =$

$= 3x^2 \cdot (1-2x) \cdot (3x^2+4x) + x^3 \cdot (-2) \cdot (3x^2+4x) + x^3 \cdot (1-2x) \cdot (3 \cdot 2x + 4),$

23) $$\left(\frac{\sin(\cos(x^2))}{\exp_4\left(tg\left(\frac{1}{x^2}\right)\right)}\right)' = \frac{[\sin(\cos(x^2))]' \cdot \exp_4\left(tg\left(\frac{1}{x^2}\right)\right) - \sin(\cos(x^2)) \cdot \left[\exp_4\left(tg\left(\frac{1}{x^2}\right)\right)\right]'}{\left[\exp_4\left(tg\left(\frac{1}{x^2}\right)\right)\right]^2}$$

ahol $[\sin(\cos(x^2))]' = \sin'(\cos(x^2)) \cdot \cos'(x^2) \cdot (x^2)' = \cos(\cos(x^2)) \cdot (-\sin(x^2)) \cdot 2x$

és $\left[\exp_4\left(tg\left(\frac{1}{x^2}\right)\right)\right]' = \exp_4'\left(tg\left(\frac{1}{x^2}\right)\right) \cdot tg'\left(\frac{1}{x^2}\right) \cdot (x^2)' = \exp_4\left(tg\left(\frac{1}{x^2}\right)\right) \cdot \ln(4) \cdot \dfrac{1}{\cos^2\left(\frac{1}{x^2}\right)} \cdot 2x$.

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