

On a class of neutral equations with state-dependent delay in population dynamics

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Abstract

In the first part of this talk we consider a class of neutral equations with state-dependent delay of the form

$$\dot{x}(t) = \frac{\alpha(x(t), x(t - \tau(x(t))), \dot{x}(t - \tau(x(t)))) - \gamma(x(t))}{1 + \dot{\tau}(x(t))\alpha(x(t), x(t - \tau(x(t))), \dot{x}(t - \tau(x(t))))}, \quad (1)$$

where α, γ are continuously differentiable functions and τ is a non-negative, bounded C^2 -function.

Equations of the class (1) satisfy the assumptions of the framework in (Walther, 2011) up to one hypothesis, which we replace by an equivalent condition. As a result, it is possible to formulate a principle of linearized stability for (1).

In the second part of the talk, we apply the results on (1) to the following problem:

$$\dot{x}(t) = \frac{\beta_{t,\tau} - \tilde{\mu}_1(x(t))}{1 + \dot{\tau}(x(t))\beta_{t,\tau}},$$

where

$$\beta_{t,\tau} := \left[\tilde{b}_1(x(t - \tau)) + b_2(x(t - \tau)) \frac{\dot{x}(t - \tau) + \tilde{\mu}_1(x(t - \tau))}{1 - \dot{\tau}(x(t - \tau))\dot{x}(t - \tau)} \right] e^{-\mu_0 \tau(x(t))}.$$

This equation expresses the dynamics of a population of mature individuals and has been derived in (Barbarossa, 2012) from an age-structured Gurtin-MacCamy model for population dynamics (Gurtin, 1974).

References

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