ULAM–HYERS STABILITY OF FUNCTIONAL EQUATIONS.

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In 1940, during a talk before the Mathematics Club of the University of Wisconsin, S.M. Ulam proposed the following problem:

Given a group $(G, \diamond)$ *and a metric group* $(G', *)$ *with metric* ρ *and given* $\varepsilon > 0$*, does there exist a* $\delta > 0$ *such that, if* $f : G \to G'$ *satisfies the condition*

$$\rho(f(x \diamond y), f(x) * f(y)) \leq \delta \qquad \textit{for all} \qquad x, y \in G$$

then a homomorphism $g : G \to G'$ *exists such that*

$$\rho(f(x), g(x)) \leq \varepsilon \qquad \textit{for all} \qquad x \in G?$$

In the affirmative case, we say that the functional equation of the homomorphisms, i.e., $\phi(x \diamond y) = \phi(x) * \phi(y)$, is **stable**.

The first positive result has been proved in 1941 by D.H. Hyers, when G and G' are the additive groups of Banach spaces.

We intend to present the evolution of Ulam's problem and its generalizations to various functional equations. In particular, in the case of homomorphisms, we discuss the algebraic properties of the groups (or semigroups) involved which imply stability.

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