Preservation of bifurcations under Runge-Kutta methods

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Abstract. We prove that fold, cusp and Bogdanov-Takens bifurcations of \( N \)-dimensional, continuous-time systems persist under Runge-Kutta methods. Compact formulae for the computation of the discretized normal form coefficients and critical generalized eigenvectors are derived.

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1. Introduction

Consider a continuous-time dynamical system depending on parameters

\[ \dot{x}(t) = f(x(t), \alpha), \]  

(1.1)

where \( f \in C^k(\Omega \times \Lambda, \mathbb{R}^N) \) with open sets \( 0 \in \Omega \subset \mathbb{R}^N, 0 \in \Lambda \subset \mathbb{R}^p, k \geq 1 \) sufficiently large, \( N \geq 1, p = 1 \) or 2. A common task in mathematical analysis is to understand the dynamics generated by the vector field (1.1). To accomplish this, we can appeal to one-step methods, which approximate the evolution operator by a discrete-time system (at previously fixed step-size)

\[ x \mapsto g(x, \alpha), \]  

(1.2)

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