Maximum Principles, Boundary Value Problems and Stability for First Order Delay Equations with Oscillating Coefficient*

Alexander Domoshnitsky

Ariel University Center of Samaria, Ariel 44837, Israel

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Abstract. On the basis of the maximum principles various assertions on unique solvability of boundary value problems, positivity of the Green’s function of the generalized periodic problem and stability for delay equation with oscillating coefficient are proposed.

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1. Definitions of Maximum Principles

The theory of the delay differential equations had been started with the equation

\[ x'(t) + p(t)x(t-\tau(t)) = 0, \quad t \in [0, \omega], \quad (1.1) \]

where

\[ x(s) = \varphi(s) \quad \text{for} \quad s < 0, \quad (1.2) \]

and \( \varphi \) is a corresponding continuous function which is called an initial function. Note that we have to add the equality (1.2) to equation (1.1) in order to define what must be set instead of \( x(t-\tau(t)) \) when \( t-\tau(t) < 0 \). The problem to define a homogeneous object is the crucial one. If equation (1.1) is studied for all possible continuous initial functions \( \varphi \), then the space of solutions of this equation becomes infinite-dimensional and there is no a direct connection between maximum principles and problems of

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