Exponential expansion of the solution of a half-linear differential equation

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Abstract. In this paper, exponential expansions of the solution are investigated for a half-linear initial value problem $y'' |y'|^{p-1} - y |y|^{p-1} = 0$ with initial conditions $y(0) = 0$, $y'(0) = 1$ or $y(0) = 1$, $y'(0) = 0$, where $p > 0$. The exponential expansions of the solution of the two nonlinear problems are investigated. By using recursive formulas we determine the coefficients of the exponential series and we provide formulas between the solutions of the two nonlinear initial value problems.

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1. Preliminaries

Our goal is to study the asymptotic solution of the half-linear differential equation

$$y'' |y'|^{p-1} - y |y|^{p-1} = 0,$$  \hspace{1cm} (1.1)

where $p$ is a positive real number, and solutions are subjected to the initial conditions

$$y(0) = 0, \ y'(0) = 1 \quad \text{(A)}$$

or

$$y(0) = 1, \ y'(0) = 0. \quad \text{(B)}$$

We note that equation (1.1) is linear with $p = 1$. Equation (1.1) is used to call half-linear one since it preserves just half of the properties which characterize linearity.