Periodicity in Neutral Functional Differential Equations by Direct Fixed Point Mapping

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Abstract. Burton-Kirk’s fixed point theorem or degree theory is used to study the existence of periodic solutions in neutral functional differential equations by constructing a homotopy which is a combination of a contraction mapping and compact mapping. The construction of such a homotopy is very difficult in practice for nonlinear equations. In this paper, we use the direct fixed point mapping technique to link the homotopy to the right hand side of the equation directly and avoid those difficulties.

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1. Introduction

We consider the system of neutral functional differential equations

\[
\frac{d}{dt} \left( x(t) - \int_{0}^{\infty} [dE(s)]x(t - s) \right) = F(t, x_t)
\]

(1.1)

where \( x(t) \in R^n \), \( F : R \times C \rightarrow R^n \) is continuous with \( C \) being the Banach space of bounded continuous functions \( \phi : (-\infty, 0] \rightarrow R^n \) with the supremum norm \( \| \cdot \| \) and \( F(t, \phi) \) is \( T \)-periodic in \( t \) for each \( \phi \in C \). Here \( E : R^+ \rightarrow R^{n \times n} \) is continuous to the left and of bounded variation on \( R^+ \). The assumption on \( E \) allows for \( \int_{0}^{\infty} [dE(s)]x(t - s) \) to include such forms as

\[
Dx(t) + Kx(t - r) + \int_{-\infty}^{t} G(t - s)x(s)ds
\]

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