Exponential Decay in Integrodifferential Equations with Nonlocal Conditions

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Abstract. We study the existence, uniqueness, and exponential decay of solutions for a semi-linear integrodifferential equation with a nonlocal initial condition

$$u'(t) = Au(t) + \int_0^t F(t-s)Au(s)ds + f(t, u(t)), \quad t \geq 0,$$

$$u(0) = \int_0^\infty g(s)u(s)ds + u_0,$$

in a Banach space $X$, with $A$ the generator of a strongly continuous semigroup. The nonlocal condition can be applied in physics with better effect than the “classical” Cauchy problem $u(0) = u_0$ since more measurements at $t \geq 0$ are allowed. The variation of constants formula for solutions via a resolvent operator and the iteration techniques are used in the study.

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