Taylor3-2c.tex, 160123.

$f(x,y,z)$ 2-rendű Taylor-polinomja

$$f:\mathbb{R}^3\to\mathbb{R}\ ,\quad \boxed{f(x,y,z)=\frac{x^2}{z}\cdot\exp\left(y^3\right)\ ,\quad \underline{a}=(2,1,5)}\ ,$$

$$f(x,y,z)=\frac{x^2}{z}\cdot\exp\left(y^3\right)\qquad\Rightarrow\qquad f\left(\underline{a}\right)=\frac{x^2}{z}\cdot\exp\left(y^3\right)=\frac{2^2}{5}e^{1^3}\approx\mathbf{2.1746},$$

$$D_xf=\frac{d}{dx}\left(\frac{x^2}{z}\cdot\exp\left(y^3\right)\right)=2\frac{x}{z}e^{y^3}\Rightarrow\left(D_xf\right)\left(\underline{a}\right)=2\cdot\frac{2}{5}e^{1^3}\approx\mathbf{2.1746}\ \text{(ok)},$$

$$D_yf=\frac{d}{dy}\left(\frac{x^2}{z}\cdot\exp\left(y^3\right)\right)=\frac{3x^2y^2}{z}e^{y^3}\Rightarrow\left(D_yf\right)\left(\underline{a}\right)=\frac{3\cdot2^2\cdot1^2}{5}e^{1^3}\approx\mathbf{6.5239}$$

$$D_zf=\frac{d}{dz}\left(\frac{x^2}{z}\cdot\exp\left(y^3\right)\right)=\frac{-x^2}{z^2}e^{y^3}\quad\Rightarrow\quad\left(D_zf\right)\left(\underline{a}\right)=\frac{-2^2}{5^2}e^{1^3}\approx-\mathbf{0.4349},$$

$$(D^2_{xx})f=\frac{d^2}{dx^2}\left(\frac{x^2}{z}\cdot\exp\left(y^3\right)\right)=\frac{2}{z}e^{y^3}\quad\Rightarrow\quad\left(D^2_{xx}f\right)\left(\underline{a}\right)=\frac{2}{5}e^{1^3}\approx\mathbf{1.0873}$$

$$(D^2_{yy})f=\frac{d^2}{dy^2}\left(\frac{x^2}{z}\cdot\exp\left(y^3\right)\right)=3x^2\frac{y}{z}e^{y^3}\left(3y^3+2\right)\quad\Rightarrow$$

$$\Rightarrow\quad\left(D^2_{yy}f\right)\left(\underline{a}\right)=3\cdot2^2\cdot\frac{1}{5}e^{1^3}\left(3\cdot1^3+2\right)\approx\mathbf{32.619}$$

$$(D^2_{zz})f=\frac{d^2}{dz^2}\left(\frac{x^2}{z}\cdot\exp\left(y^3\right)\right)=2\frac{x^2}{z^3}e^{y^3}\Rightarrow\left(D^2_{zz}f\right)\left(\underline{a}\right)=2\cdot\frac{2^2}{5^3}e^{1^3}\approx\mathbf{0.1739}$$

$$\left(D^2_{xy}\right)f=\frac{d^2}{dxy}\left(\frac{x^2}{z}\cdot\exp\left(y^3\right)\right)=6x\frac{y^2}{z}e^{y^3}\Rightarrow\left(D^2_{xy}f\right)\left(\underline{a}\right)=6\cdot2\frac{1^2}{5}e^{1^3}\approx\mathbf{6.5239}$$

$$\left(D^2_{xz}\right)f=\frac{d^2}{dxz}\left(\frac{x^2}{z}\cdot\exp\left(y^3\right)\right)=-2\frac{x}{z^2}e^{y^3}\Rightarrow\left(D^2_{xz}f\right)\left(\underline{a}\right)=-2\frac{2}{1^2}e^{1^3}\approx-\mathbf{10.873},$$

$$\left(D^2_{yz}\right)f=\frac{d^2}{dyz}\left(\frac{x^2}{z}\cdot\exp\left(y^3\right)\right)=-3x^2\frac{y^2}{z^2}e^{y^3}\quad\Rightarrow\left(D^2_{yz}f\right)\left(\underline{a}\right)=-3\cdot2^2\frac{1^2}{5^2}e^{1^3}\approx-\mathbf{1.3048},$$

Tehát :

$$\left(T^2_{\underline{a}},f\right)(x,y,z)\approx2.1746+2.1746(\mathbf{x-2})+6.5239(\mathbf{y+1})-0.4349(\mathbf{z-5})+$$

$$+\frac{1}{2!}\left(1.0873(\mathbf{x-2})^2+32.619(\mathbf{y+1})^2+0.1739(\mathbf{z-5})^2\right)+$$

$$+\frac{2}{2!}\left(6.5239(\mathbf{x-2})^1(\mathbf{y+1})^1-10.873(\mathbf{x-2})^1(\mathbf{z-5})^1-1.3048(\mathbf{y+1})^1(\mathbf{z-5})^1\right)$$