

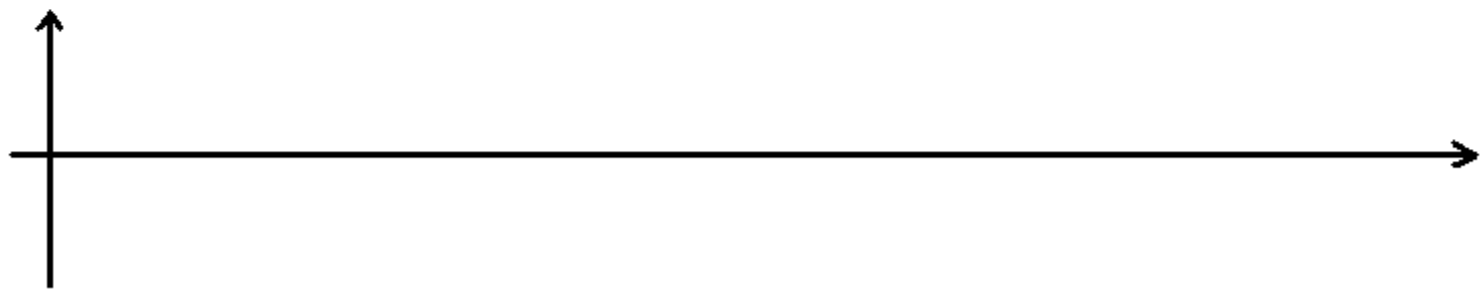
Gyökközelítés intervallum-felezéssel

Szalkai István

Veszprém,

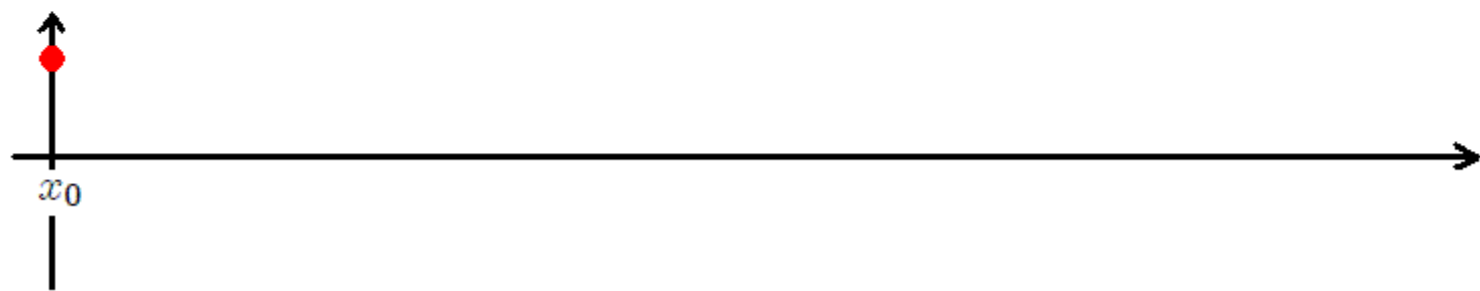
2018.10.30.

$$x + 1 = \operatorname{tg}(x) \quad \Longrightarrow \quad g(x) := x + 1 - \operatorname{tg}(x) = 0$$



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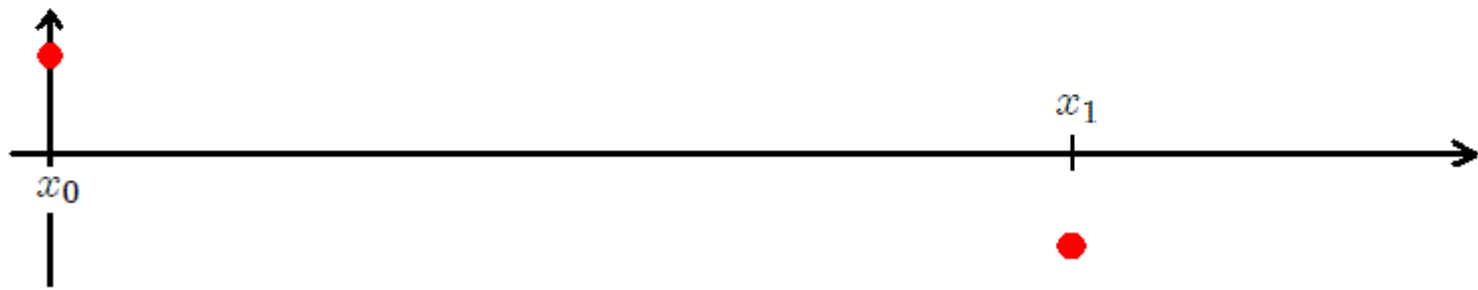
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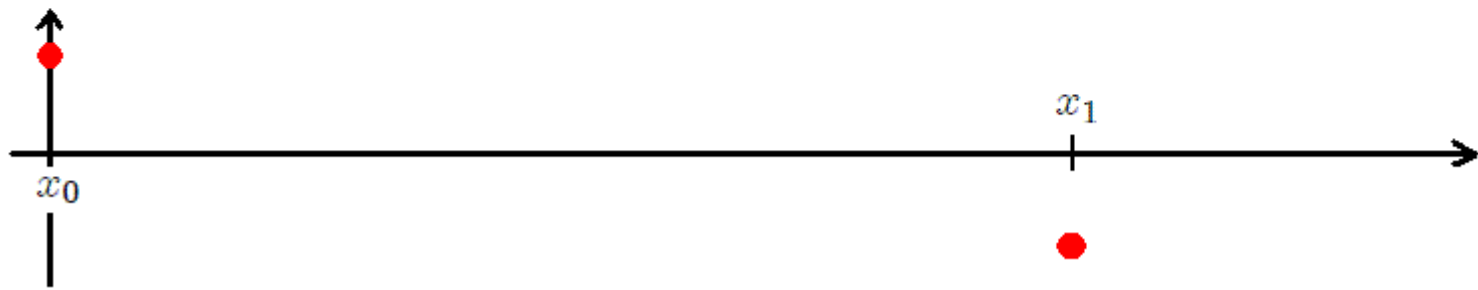
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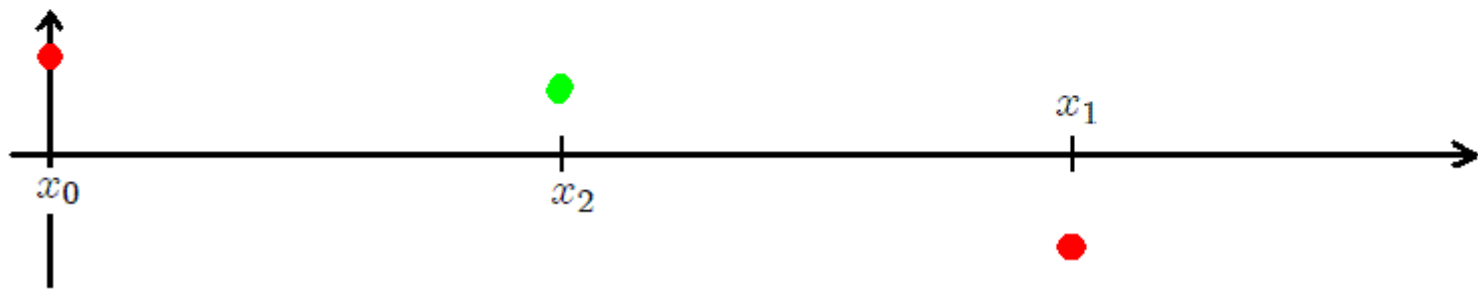


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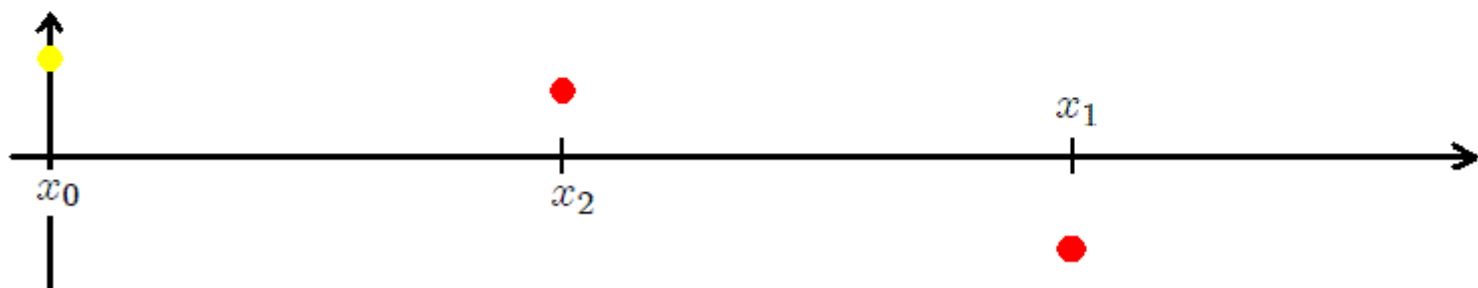


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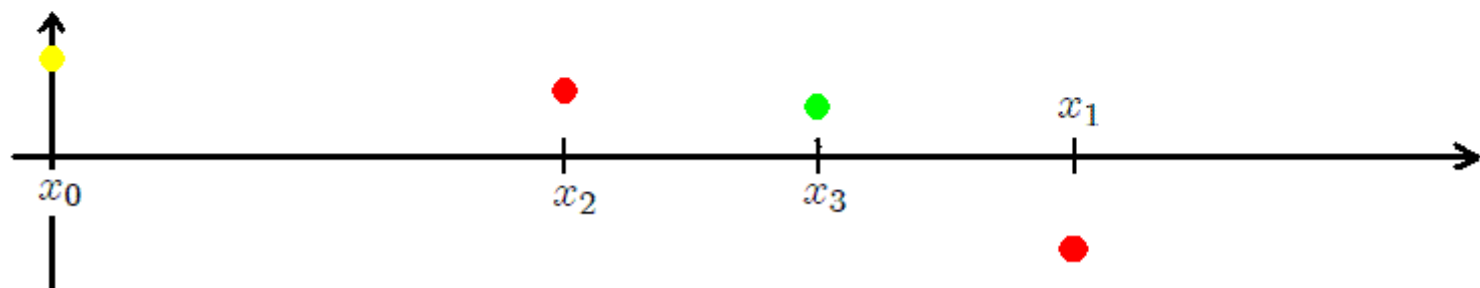
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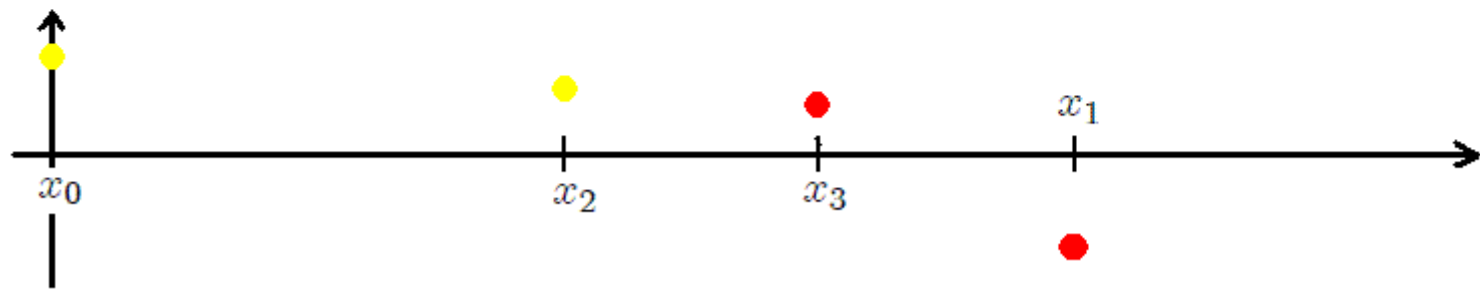
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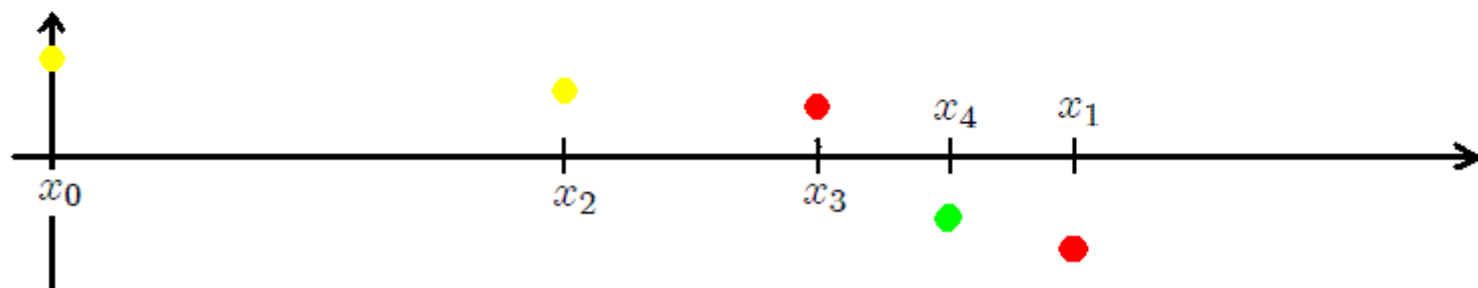
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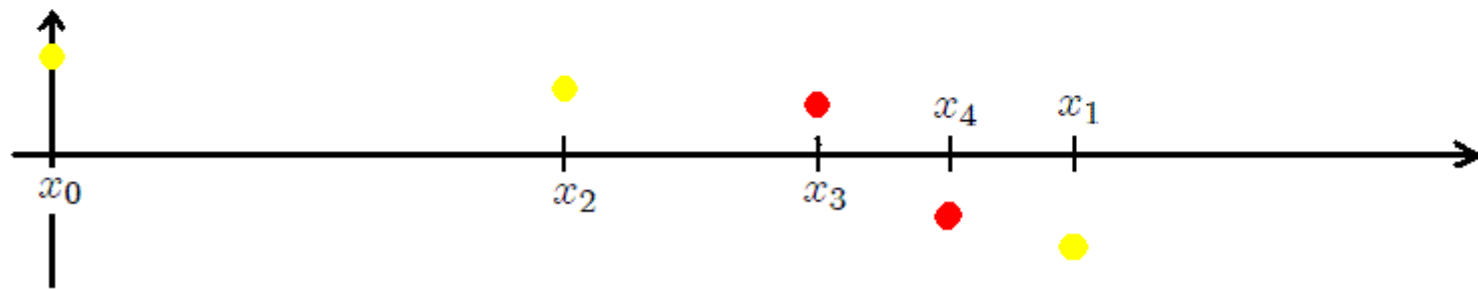
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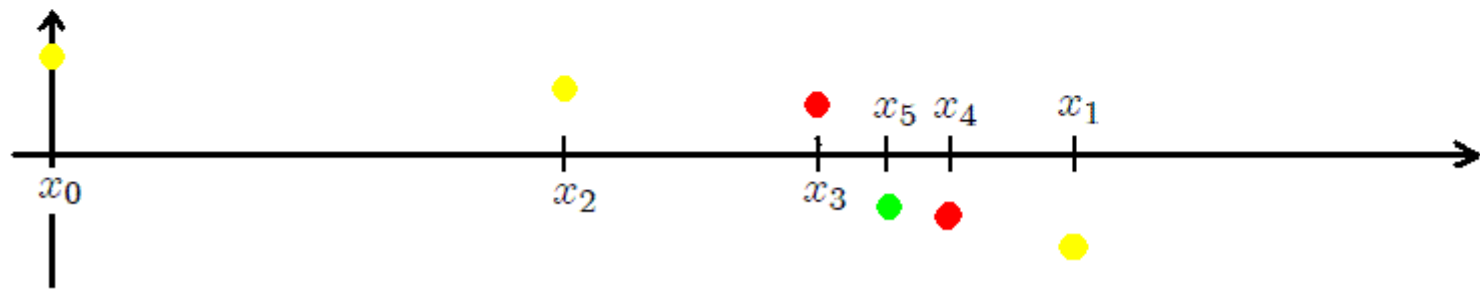
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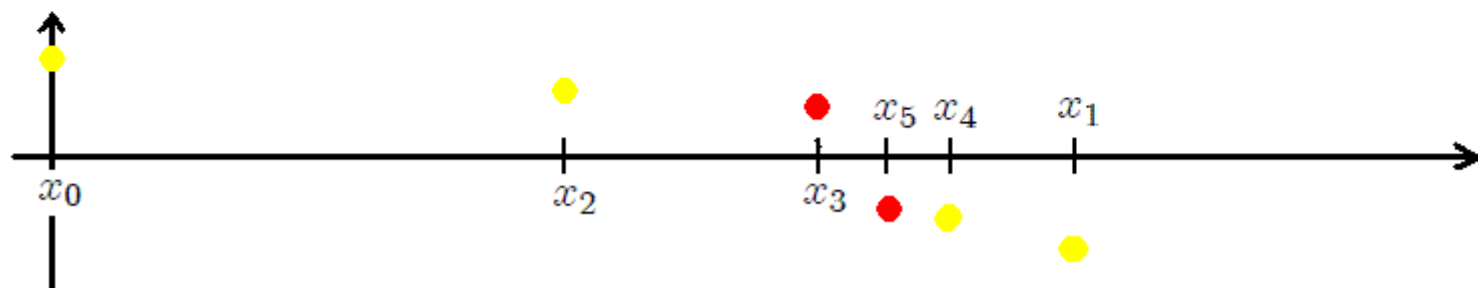
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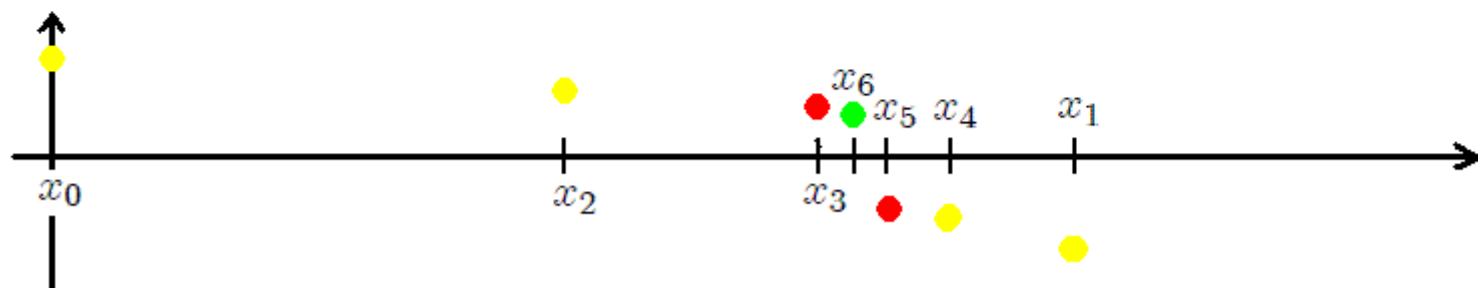
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$$x_6 = \frac{x_3 + x_5}{2} \approx 1.09375, \quad g(x_6) \approx 0.16 > 0$$



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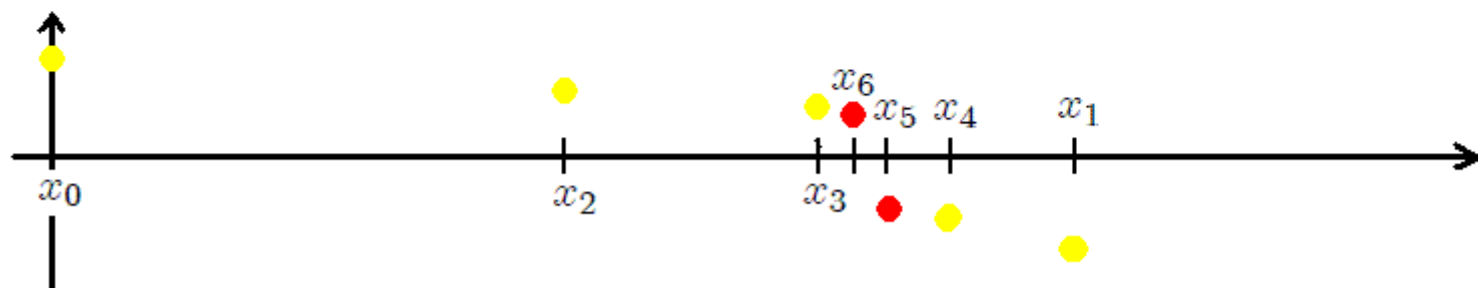
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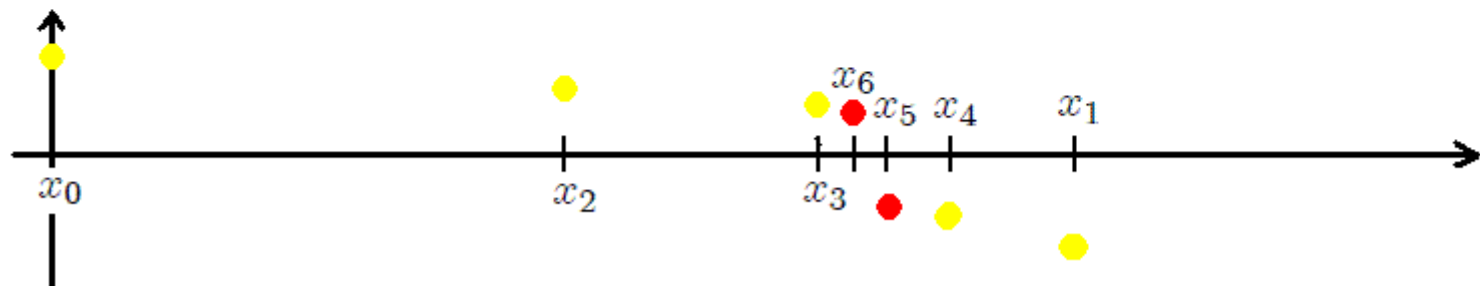
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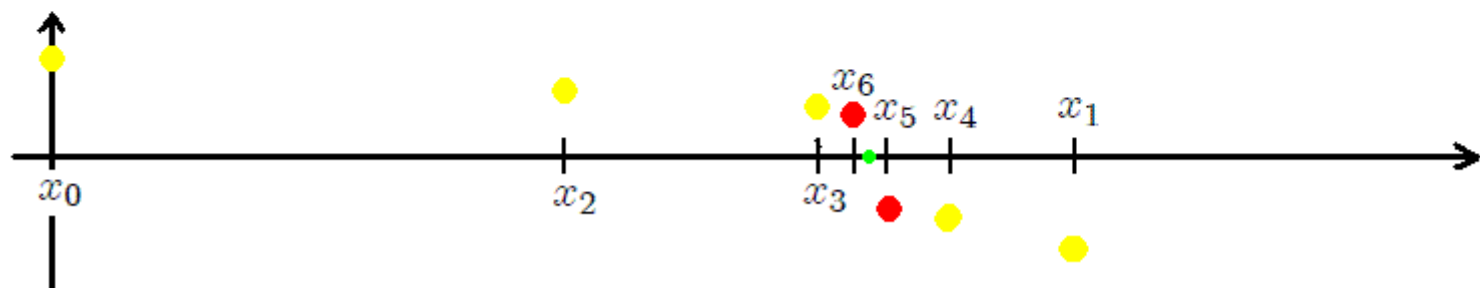
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tehát 5 lépés után $x^* \approx \frac{x_6 + x_5}{2} \approx 1.1156$,

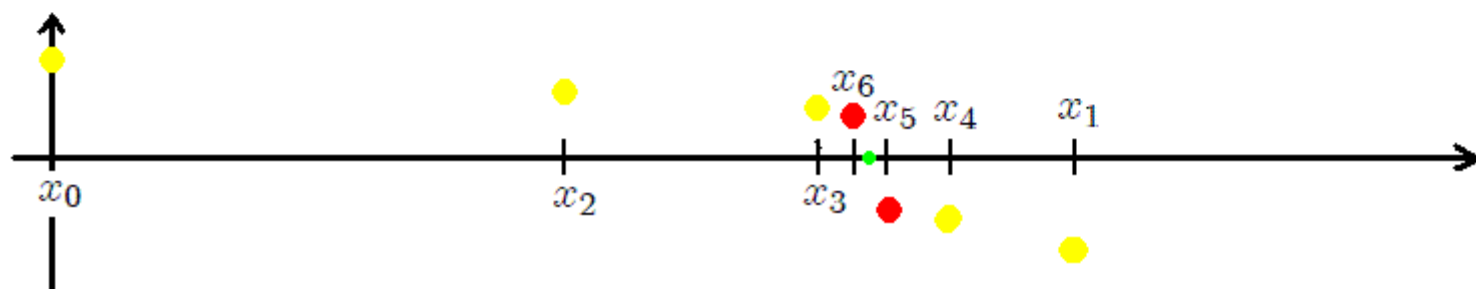


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$$\text{Megjegyzés:} \quad \frac{|x_6 - x_5|}{2} = \frac{|b - a|}{2^6}.$$

$$\text{További 5 lépés (} n = 10 \text{) után} \quad x^* \approx 1.132715,$$

$$\text{a hiba pedig} \quad \varepsilon \leq \frac{|b - a|}{2^{11}} \approx 0.000684.$$

