Példa numerikus integrálásra

http://math.uni-pannon.hu/~szalkai/NumInt-pl-181101.pdf

Trapézformula: $n = 10$, $b - a = 1$,

$$\int_3^4 \frac{x^2}{\ln(x)} dx \approx \frac{b-a}{n} \cdot \left(\frac{f(a)}{2} + f(x_1) + f(x_2) + ... + f(x_{n-1}) + \frac{f(b)}{2} \right) =$$

$$= \frac{1}{10} \cdot \left(\frac{1}{2} \cdot \frac{3^2}{\ln(3)} + \frac{3.1^2}{\ln(3.1)} + \frac{3.2^2}{\ln(3.2)} + \frac{3.3^2}{\ln(3.3)} + \frac{3.4^2}{\ln(3.4)} + \frac{3.5^2}{\ln(3.5)} + \frac{3.6^2}{\ln(3.6)} + \frac{3.7^2}{\ln(3.7)} + \right.$$

$$\left. + \frac{3.8^2}{\ln(3.8)} + \frac{3.9^2}{\ln(3.9)} + \frac{1}{2} \cdot \frac{4^2}{\ln(4)} \right) \approx \underline{9.808\,4}$$

hiba $\varepsilon \leq \frac{K \cdot (b-a)^3}{12n^2} = \frac{K \cdot (4-3)^3}{12 \cdot 10^2} = \frac{K}{1200}$

Simpson formula, $n = 10$, $b - a = 1$,

$$\int_3^4 \frac{x^2}{\ln(x)} dx \approx \frac{b-a}{3n} \cdot \Big(f(a) + 4f(x_1) + 2f(x_2) + 4f(x_3) + ... \quad + 2f(x_{n-2}) + 4f(x_{n-1}) + f(b) \Big) =$$

$$= \frac{1}{30} \cdot \left(\frac{3^2}{\ln(3)} + \frac{4 \cdot 3.1^2}{\ln(3.1)} + \frac{2 \cdot 3.2^2}{\ln(3.2)} + \frac{4 \cdot 3.3^2}{\ln(3.3)} + \frac{2 \cdot 3.4^2}{\ln(3.4)} + \frac{4 \cdot 3.5^2}{\ln(3.5)} + \frac{2 \cdot 3.6^2}{\ln(3.6)} + \frac{4 \cdot 3.7^2}{\ln(3.7)} + \frac{2 \cdot 3.8^2}{\ln(3.8)} + \right.$$

$$\left. + \frac{4 \cdot 3.9^2}{\ln(3.9)} + \frac{4^2}{\ln(4)} \right) \approx \underline{9.807\,8}$$

hiba $\varepsilon \leq \frac{M \cdot (b-a)^5}{180n^4} = \frac{M \cdot (4-3)^5}{180 \cdot 10^4} = \frac{M}{1800\,000}$

Deriváltak:

$$\frac{d}{dx}\left(\frac{x^2}{\ln x} \right) = \frac{2x \cdot \ln x - x^2 \cdot \frac{1}{x}}{\ln^2 x} = \frac{x \cdot (2\ln x - 1)}{\ln^2 x} ,$$

$$\frac{d^2}{dx^2}\left(\frac{x^2}{\ln x} \right) = \frac{d}{dx}\left(\frac{x \cdot (2\ln x - 1)}{\ln^2 x} \right) =$$

$$= \frac{\left[1 \cdot (2\ln x - 1) + x \cdot \left(\frac{2}{x} - 0 \right) \right] \cdot \ln^2 x - [x \cdot (2\ln x - 1)] \cdot 2\ln x \cdot \frac{1}{x}}{\ln^4 x} =$$

$$= \frac{[2\ln x - 1 + 2] \cdot \ln^2 x - (2\ln x - 1) \cdot 2\ln x}{\ln^4 x} = \frac{2\ln^3 x - 3\ln^2 x + 2\ln x}{\ln^4 x} = \frac{2\ln^2 x - 3\ln x + 2}{\ln^3 x} ,$$

$$\frac{d^3}{dx^3}\left(\frac{x^2}{\ln x} \right) = \frac{d}{dx}\left(\frac{2\ln^2 x - 3\ln x + 2}{\ln^3 x} \right) = \ldots = \frac{-2\left(\ln^2 x - 3\ln x + 3 \right)}{x \ln^4 x} ,$$

. . .