

FOURTH EUROPEAN SUMMER SCHOOL IN LOGIC
CONFERENCE
EUROPEAN SUMMER MEETING
OF THE ASSOCIATION FOR SYMBOLIC LOGIC

Veszprém, 1992

The 1992 European Summer Meeting of the Association for Symbolic Logic was held in Veszprém, Hungary, from August 9 to August 15, 1992.

The meeting was organized jointly by the János Bolyai Mathematical Society, the Mathematical Institute of the Hungarian Academy of Sciences, and by the Symbolic Logics Department of the Loránd Eötvös University. Financial support was received from ASL, IUHPS/DLMPS, from the Hungarian Academy of Sciences, and from OMFB.

The Organizing Committee members were A. Hajnal (chair), L. Csirmaz (secretary), H. Andréka, J. Demetrovics, G. Fazekas, T. Gergely, F. Gécseg, I. Juhász, P. Komjáth, Ms. T. Madarász, I. Németi, I. Ruzsa, I. Sain, L. Soukup, J. Surányi, M. Szöts, L. Úry (Budapest), and A. Dragálin (Debrecen).

The Program Committee was formed by J. Baumgartner, J. van Benthem, S. Givant, M. Makkai, D. Monk, J. Paris, V. Pratt, P. Pudlak, S. Shelah (international), and A. Dragálin, T. Gergely, A. Hajnal, Ms T. Madarász, I. Németi (local).

The meeting was attended by 150 registered participants from 21 countries.

There were 13 one-hour invited lectures:

H. Andréka, *Recent trends in algebraic logic, language, and information.*

M. Creswell, *Restricted quantification.*

K. Fine, *Progressive reasoning.*

M. Foreman, *An \aleph_1 -dense ideal on ω_2 , small ultrapowers and chromatic numbers.*

D. Gabbay, *Fibred semantics for LDS.*

M. Gitik, *The singular cardinal problem.*

R. Goldblatt, *Varieties of Boolean algebras with operators.*

Yu. Gurevich, *Finite model theory.*

P. Komjáth, *Paradoxical decomposition of Euclidean spaces.*

H. Kotlarski, *Automorphisms of countable, recursively saturated models for Peano arithmetic.*

L. Levin, *Randomness and nondeterminism in computing.*

R. Maddux, *On the derivation of identities involving projection functions and direct products.*

I. Ruzsa, *Intensional logic admitting semantic value gaps.*

In addition 90 contributed papers were accepted in five parallel sections, of which 74 were actually presented at the meeting. Unfortunately, the Organizers had a limited budget, so grants were offered only in a few cases. More available grants would have made possible the attendance of several colleagues with limited financial possibilities.

László Csirmaz, secretary to the meeting

Invited addresses

H. ANDRÉKA, *Recent trends in algebraic logic, language and information.*

Mathematical Institute, Hungarian Academy of Sciences, 1053 Budapest, Hungary.

This lecture intends to be an informal overview of recent developments which underly the cooperation of the Budapest group with the Berkeley group as well as with the European Foundation for Logic,

ISTVÁN JOÓ, MIKLÓS HORVÁTH and ISTVÁN SZALKAI, *Continuity of operators from logical point of view.*

Department of Analysis, Eötvös University, Budapest, Hungary (Joó and Horváth).

Department of Mathematics, University of Veszprém, Veszprém, Hungary (Szalkai).

All the linear operators on Banach spaces that have been studied in analysis until now (not using AC but if we define both the Banach spaces and the operators itself by first-order formulae) are *continuous*. Now the question is the following:

“Are all the first-order formula definable linear operators on first-order formula definable Banach spaces continuous?”

The answer in L is no. On the other hand, M. Ajtai proved in [1] and [2] that adding a Cohen generic real to any model of ZFC, then in the generic extension, every linear operator between Banach spaces (all of them are first-order formula definable) is continuous. Further, when the formulae above satisfy certain absoluteness conditions, we can derive absolute theorems: the operators defined by these kind of formulae are continuous in all models, not only in the generic extensions.

In our paper we give generalizations of these and Ajtai's other results. Further, we give some applications. We derive absolute theorems in analysis using mathematical logical methods. For example, we deduce the following theorem using the above results.

THEOREM (in ZFC). *There is no sequence $\langle a_n : n \in \mathbb{Z} \rangle$ consisting of positive numbers, uniquely definable by a first-order formula s.t. $\lim_{|n| \rightarrow \infty} a_n = 0$ and for any $f \in L^1(0, 2\pi)$ we have $|c_n(f)| \leq c_f a_n$ ($n \in \mathbb{Z}$) for some constant $c_f > 0$ not depending on $n \in \mathbb{Z}$.*

Here $c_n(f) = \int_0^{2\pi} f(x)e^{-inx} dx$ is the n th Fourier coefficient.

REFERENCES

- [1] M. AJTAI, *On the boundedness of definable linear operators*, *Periodica Mathematica Hungarica*, vol. 5 (1974), pp. 343–352.
- [2] ———, *Definable Banach spaces*, Ph.D. Thesis, Eötvös University, Budapest, 1974.

I. JUHÁSZ, *On the weight-spectrum of a compact space.*

Department of Mathematics, Kansas State University, Manhattan, Kansas 66506.

Mathematics Institute, Hungarian Academy of Science, 1364 Budapest, Hungary.

The weight-spectrum $Sp(w, X)$ of a space X is the set of weights of all infinite closed subspaces of X . We prove that if $\kappa > \omega$ is regular and X is compact T_2 with $w(X) \geq \kappa$ then some λ with $\kappa \leq \lambda \leq 2^{<\kappa}$ is in $Sp(w, X)$. Under CH this implies that the weight spectrum of a compact space cannot omit ω_1 , and thus solves problem 22 of [M]. Also, it is consistent with $2^\omega = c$ being anything it can be that every countable closed set T of cardinals less than c with $\omega \in T$ satisfies $Sp(w, X) = T$ for some separable compact LOTS X . This shows the independence from ZFC of a conjecture made in [AT]. This research was supported by OTKA grant number 1908.

REFERENCES

- [AT] A. V. ARHANGEL'SKII and V. V. TKAČUK, *Calibers and point-finite cellularity of the space $C_p(X)$* , and some questions of S. Gul'ko and M. Hušek, *Topology and its Applications*, vol. 23 (1986), pp. 65–74.
- [M] J. D. MONK, *Cardinal functions on Boolean algebras*, Birkhäuser Verlag, Basel, 1990.

VLADIMIR G. KANOVEI, *Some undecidability in internal set theory.*

Moscow State University, Moscow 107392, Russia.

In [1977] Edward Nelson gave a new formulation of nonstandard analysis, known as Internal Set Theory IST. This theory extends ZFC by adding the standardness predicate *st* and three new axioms, the Idealization (I), the Standardization (S), and the Transfer (T), governing the interconnections between standard and nonstandard sets. The following *Extension Principle* is also widely used in many applications, see, e.g., Diener and Stroyan [1988]:

(E) For all standard X and Y ,

$$\forall^{st}x \in X \exists y \in Y \Phi(x, y) \rightarrow \exists \tilde{y}: X \mapsto Y \forall^{st}x \in X \Phi(x, \tilde{y}(x)).$$

It is known (see Diener and Stroyan [1988]) that (E) is a theorem of IST for all bounded formulas Φ , that is, those having all the occurrences of the predicate *st* only through the quantifiers of type $\exists^{st}u \in v$ and $\forall^{st}u \in v$.

THEOREM. (E) is undecidable in IST.

This means that (1) $\text{Con IST} \rightarrow \text{Con IST} + (E)$ for all Φ and (2) $\text{Con IST} \rightarrow \text{Con IST} + \neg(E)$ for some explicitly defined Φ .

There are a number of other undecidable hypotheses in IST; see Kanovei [1991].

REFERENCES

E. NELSON [1977], *Internal set theory: a new approach to nonstandard analysis*, *Bulletin of the American Mathematical Society*, vol. 83, pp. 1165–1198.

F. DIENER and K. D. STROYAN [1988], *Syntactical methods in infinitesimal analysis*, *Nonstandard analysis and its applications* (N. Cutland editor), Cambridge University Press, London, pp. 258–281.

V. G. KANOVEI [1991], *Undecidable hypotheses in Edward Nelson's internal set theory*, University of Amsterdam, ITLI X-91-16.

M. KOJMAN and S. SHELAH, *Universal abelian groups*.

Mathematical Institute, Hebrew University, Jerusalem 91904, Israel.

This paper concerns the general question in model theoretical set theory of the existence of a universal model in a class of models (namely, one in which all other members of the class of the same cardinality are embedded). Using combinatorial tools which were developed by Shelah, it became possible to prove that existence or nonexistence of universal models in various classes may be deduced for many cardinalities from cardinal arithmetic. We investigate various known classes of abelian groups (separable p -groups, torsion-free groups, slender groups) with respect to the existence of a universal object in the class. We prove, for example,

THEOREM. If $\mu^+ < \lambda = \text{cof}(\lambda) < \mu^{\aleph_0}$ and $\lambda > 2^{\aleph_0}$, then there is no universal separable p -group [torsion free, slender] in cardinality λ .

ÁGNES KURUCZ, *Connections between axioms of set theory and basic theorems of universal algebra*. Mathematical Institute, Hungarian Academy of Sciences, Budapest, P.O. Box, 127 H-1364, Hungary.

The problems investigated here fit into the field which is called after S. G. Simpson *reverse mathematics* or *inverse set theory*. In this field one tries to find out what axioms of set theory are implied by well-known theorems of mathematics. The first "reverse" questions concerning universal algebra were put and partly answered by G. Grätzer. Problem 31 in his book [7] asks whether Birkhoff's variety theorem (that is, the model-class of the equations holding in a class of algebraic structures is equal to the class obtained by taking homomorphic images of substructures of direct products of some members of the class in question) is equivalent to the Axiom of Choice. As it was shown in [3], the answer is no: Birkhoff's theorem can be derived already in ZF.

In the paper we show that it is a *weakening* of Regularity which plays the crucial role. In the first part of the paper a permutation model is constructed which proves that Birkhoff's theorem cannot be derived in $\text{ZF} \setminus \{\text{Regularity}\}$, even if one adds Regularity for Finite sets to it. Since we also prove that, on the basis of $\text{ZF} \setminus \{\text{Regularity}\}$, Birkhoff's theorem is equivalent to the Collection schema (which, in turn, is implied by Regularity), our permutation model shows the independence of the Collection schema of the axiom set above. The axiom set $\text{ZF} \setminus \{\text{Regularity}\} + \text{Collection}$ seems to be important in universal algebra. Moreover, in the second part of the paper we prove (a) that (in $\text{ZF} \setminus \{\text{Regularity}\}$) some well-known theorems of universal algebra are equivalent to Collection, and (b) that—on the same basis—some more "intricate" ones are equivalent to Collection plus Choice that is, to "Choice for sets of non-empty classes".

REFERENCES

[1] H. ANDRÉKA, P. BARMEISTER, and I. NÉMETHI. *Quasivarieties of partial algebras—a unifying approach towards a two-valued model theory for partial algebras*, *Studia Scientiarum Mathematicarum Hungarica*, vol. 16 (1981), pp. 325–372.

[2] H. ANDRÉKA and I. NÉMETHI, *Does $\text{SP K} \supseteq \text{PS K}$ imply the axiom of choice?*, *Commentationes Mathematicae Universitatis Carolinae*, vol. 12 (1980), pp. 699–706.

[3] ———, *HSP K is equational class, without the axiom of choice*, *Algebra Universalis*, vol. 13 (1980), pp. 164–166.

[4] U. FELGNER, *Models of ZF-set theory*, Lecture Notes in Mathematics, vol. 223, Springer-Verlag, Berlin and New York, 1971.

[5] H. M. FRIEDMAN, S. G. SIMPSON, and R. I. SMITH, *Countable algebra and set existence axioms*, *Annals of Pure and Applied Logic*, vol. 25 (1983), pp. 141–181.