EÖTVÖS LORÁND TUDOMÁNYEGYETEM
FACULTY OF NATURAL SCIENCES

THIRD CONFERENCE OF PROGRAM DESIGNERS

(July 1–3, 1987)

Ed. by A. Iványi

Budapest, 1987

FOREWORD

From 1972, the beginning of the special education for computer specialists in Hungary, about 850 batchelors and 300 masters of science graduated at Eötvös Loránd University /ELU/. Up to the present seven of them received the Ph. D. degree.

Since 1981, ELU has organized every year a conference for young Hungarian and foreign programmers and mathematicians.

In 1985 ELU commemorated the 350-th anniversary of its foundation. In the framework of the jubilee celebrations our university has organized a series of events, among others "The First Conference of Program Designers" /PD'85/. More than hundred specialists among them 89 program designers participated in the work of the conference.

Continuing the series of the previous conferences we organized "The Second Conference of Program Designers" /Budapest, July 8-9, 1986/ and "The Third Conference of Program Designers" /Budapest, July 1-3, 1987/.

The program of PD'87 consisted of two invited and twenty six submitted lectures, projection of videofilms, chess and tennis tournaments. According to the propositions of the program designers the invited survey lectures were delivered in Hungarian.

This book contains 34 papers. The authors of the papers represent 11 countries. The main topics are methodology of programming, theory of algorithms, probability theory and numerical analysis. The majority of the materials is written in English, the remaining part in Russian /the titles are summarized in English, in Hungarian and in Russian/.

SEE YOU AGAIN ON THE NEXT PROGRAM DESIGNER CONFERENCE!

Budapest, July 3, 1987

Antal Iványi

PROVING THEOREMS IN ANALYSIS USING MATHEMATICAL LOGICAL METHODS

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The following result was proved by M.Ajtai :

__THEOREM__ 1 (Ajtai, [A1], [A2]) Let M be a transitive ZFC model and the formula $\varphi(X, \varrho, F, c)$ define a complete topological space with pseudometric ϱ and a rational convex functional F on it from the parameter $c \in M$. If G is a Cohen-generic real over M and X, ϱ, F satisfy φ in M[G] then F is continuous in M[G]. ■

It is well-known that there exists an infinite dimensional topological vector space and a non-continuous linear operator on it, both definable by a first order formula assuming V=L. Ajtai's theorem has a simple consequence:

__COROLLARY__ 2. Suppose that the formulae $\varphi_1, \varphi_2, \psi$ define in ZFC complete metric spaces B_1, B_2 and a linear operator L between them. Then, in M[G], L is continuous. ■

The following observation is the starting point of our results: If M and N are ZFC-models, $M \subset N$ and the formula φ defines (in ZFC) a complete metric space and $\langle X^M, \varrho^M \rangle \in M$ and $\langle X^N, \varrho^N \rangle \in N$ are such that $M \models \varphi(X^M)$ and $N \models \varphi(X^N)$ then $\langle X^M, \varrho^M \rangle$ is a metric space over R^M which is a dense subset of R^N. Denote by $(X^M)^N_c$ the complete metric space generated by X^M in N. Then the following absoluteness statement holds:

__STATEMENT__ Let φ_1 and φ_2 and ψ be formulae such that for $i = 1, 2$

$$ZFC \vdash "(\exists! B)(\varphi_i(B)) \& (\forall B)[\varphi_i(B) \Rightarrow B = \langle X, \varrho \rangle \& \langle X, \varrho \rangle \text{ is a complete metric space}]"$$

and

$$ZFC \vdash "(\exists! z)(\psi(z)) \& (\forall B_1, B_2, z)[\varphi_1(B_1) \& \varphi_2(B_2) \& \psi(z) \Rightarrow z : B_1 \to B_2 \text{ is a rational convex operator}]".$$

Suppose further that for every ZFC-models M and N, $M \subset N$ the following implication holds:

if L, B_1, B_2 are in M and $M \models \varphi_1(B_1) \& \varphi_2(B_2) \& \psi(L)$ and L', B'_1, B_2 are in N and $N \models \varphi_1(B'_1) \& \varphi_2(B'_2) \& \psi(L')$ then there are t_1, t_2 in N such that

$N \models$ " $t_i : (B_i)^N_c \to B'_i$ is an isometric imbedding for $i = 1, 2$, and $t_2(L(x)) = L'(t_1(x))$ for $x \in X_1$".

Then for every ZFC-model M

$$M \models \text{"}(\forall B_1, B_2, L)[\varphi_1(B_1) \& \varphi_2(B_2) \& \psi(L) \Rightarrow L \text{ is continuous}]\text{"}. \blacksquare$$

Using this observation we get absolute theorems in analysis. First we verify the assumptions of the Statement 3. for some metric spaces:

STEP A The Banach space $C[0,1]$ with the supremum-norm satisfies the first line of the Statement 3.

STEP B The property " A is a set of Lebesgue measure null"

STEP C In order to define the spaces $L^p(0,1)$, use the following known facts. A function $f:(0,1) \to \mathbb{R}$ is measurable if and only it is the pointwise a.e. limit of continuous functions. Define the metric $\varrho_s(f,g) := \inf\{a + |(|f-g|>a)| : a > 0\}$. Then ϱ_s generates the convergence in measure which is weaker than the a.e.convergence. Formalizing the above facts we get that the space $\langle L^0(0,1), \varrho_s \rangle$ satisfies the first line of Statement 3. Further for $f \in L^p(0,1)$

$$\int_{[0,1]} |f|^p = \int_{[0,\infty)} |(|f| > \alpha)| \alpha^{p-1} d\alpha$$

consequently the space $\langle L^p(0,1), \|\cdot\|_p \rangle$ also satisfies the first line of Statement 3.

STEP D The Banach space $\langle c_\infty, \|\cdot\|_\infty \rangle$ satisfies the first line of Statement 3.

Now fix a sequence $\langle a_n : n \in \omega \rangle$ of positive numbers converging to zero and define the operator $F : L^1(0,1) \to c_\infty$ by $F(f) := \langle c_n(f)/a_n : n \in \omega \rangle$ for $f \in L^1(0,1)$.

THEOREM 4 There is no sequence $\langle a_n ; n \in \omega \rangle$ of positive numbers converging to zero and definable by a first order formula such that for every $f \in L^1(0,1)$ there exists $c_f > 0$ with $|c_n(f)| \le c_f a_n$ $(n \in \omega)$. $\blacksquare$

In our paper [JHSz] we generalize the original theorem of Ajtai fornon-metrizable topological vector spaces:

THEOREM 5 [JHSz] Let V be a vector space, $V = \bigcup_{n \in \omega} V_n$ where each V_n has a complete semi-metric ϱ_n and consider the weak topology on V generated by the inclusions $V_n \subset V$. Then the analogy of Theorem 1 holds, namely if $\langle V, \tau \rangle$ is definable by a first order formula, F is a definable convex rational operator on V : then adding a Cohen-generic real to an arbitrary model M, F^N is continuous in N on V^N where $N = M[G]$. $\blacksquare$

In order to present our following result, some definitions are needed.

DEFINITION Let H be an arbitrary set. Then

a/ ${}^\omega H$ denotes the set $\{ f : f$ is a function, $\mathrm{Dom}(f) = \omega$ and $\mathrm{Ran}(f) \subseteq H \}$.

b/ If $n \in \omega$ and $h_i \in H$ for $i < n$ then we denote by $\overrightarrow{\langle h_i ; i<n \rangle}$ the element $\vec{\ell}$ of ${}^\omega H$ for which the equalities $\vec{\ell}(i) = h_i$ for $i<n$ and $\vec{\ell}(i) = h_n$ for $i \ge n$ hold.

c/ If $n \in \omega$ and $\vec{h} \in {}^\omega H$ then we denote by $\overrightarrow{(\vec{h} \restriction n)}$ the element $\vec{\ell}$ of ${}^\omega H$ for which the equalities $\vec{\ell}(i) = h(i)$ for $i < n$ and $\vec{\ell}(i) = h(n)$ for $i \ge n$ hold.

THEOREM 6 [JHSz] Let φ be a formula. Suppose that the following four statements can be proved in ZFC :

a/ For any Banach space $\langle B, F \rangle$ and T^0 , if $\varphi(B, F, T^0)$ holds, then T^0 is a function with $\mathrm{Dom}(T^0) = {}^\omega(B^*)$ and for any $\vec{A}$ from $\mathrm{Dom}(T^0)$, $T^0(\vec{A})$ is a linear mapping from B to $\mathbb{R} \cup \{+\infty\}$.

Denote then $S := \{ \vec{A} \in B^* : \mathrm{Ran}(T^0(\vec{A})) \subseteq \mathbb{R} \}$

and by T the restriction of T^{o} to S.

b/ If (B,F) and T are as above and $A_i \in B^*$ for $i < n$ where $n \in N$ is arbitrary then $T(\langle A_i : i < n\rangle^{\rightarrow}) \in B^*$.

c/ If (B,F) and T are as above, $\vec{A} \in {}^{\omega}(B^*)$ then for any $n < \omega$ and $x \in B$ the inequality $T^{o}(\vec{A})(x) > n$ holds if and only if $T((\vec{A} \upharpoonright m)^{\rightarrow})(x) > n$ for every large $m < \omega$.

d/ If (B_1,F_1) and (B_2,F_2) are Banach spaces, B_1 is a dense subspace in (B_2,F_2), $F_1 = F_2 \upharpoonright B_1$ and we have $\varphi(B_1,F_1,T_1^{o})$ and $\varphi(B_2,F_2,T_2^{o})$, and further $A_i^{(1)} \in B_1^*$ and $A_i^{(2)} \in B_2^*$ for $i < m$ where m is finite, and A_i^2 isa continous extension of $A_i^{(1)}$ onto B_2, then the functional $T_2(\langle A_i^{(2)} : i < m\rangle^{\rightarrow})$ is also a continuous extension of $T_1(\langle A_i^{(1)} : i < m\rangle^{\rightarrow})$ onto B_2.

Then the following assertion is also provable in ZFC:
If (B,F) is Banach space, $\varphi(B,F,T^{o})$ and $\vec{A}$ is in Dom (T) then $T(\vec{A})$ is continuous. ∎

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