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ON THE DEFINITIONS OF COMPUTABLE ANALYSIS

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In my paper I examine the basic definitions of computable analysis and their equivalence. Some part of these investigations can be found in the literature but as far as I know there is not a complete collection of these investigations like my present paper. Proofs are omitted because of their length.

§.1. COMPUTABLE REAL NUMBERS

DEF.1.:/29/,/35/. A real number x is recursive computable if there is a natural number $g \geq 2$ and recursive function f such that $\forall n \; 0 \leq f/n/ < g$ and $x = \sum_{i=0}^{\infty} f/i/ \cdot g^{-i}$.

DEF.2.:/60/. A real number x is called recursive real number if there is a recursive function f such that $\forall n \; 0 \leq f/n/ < n$ and $x = \sum_{i=0}^{\infty} f(i) / i!$.

DEF.3.: A real number x is recursive computable if the function $f/n/ = [nx]$ /the integer part of x/ is recursive.

DEF.4. A real number x is recursive computable if there is a recursive function f such that $\forall n \; | f(n)/n - x | < 1/n$.

DEF.5.:/1/. A function $f: Q \leftrightarrow Q$ is <u>programmable</u> if there are recursive functions g,h and k such that for every rational number $x = (r-s)/(1+t)$ $f/x/ = \left(g/r,s,t/ - h/r,s,t/ \right) / \left(1 + k/r,s,t/ \right)$ holds.

DEF.6.: A real number x is recursive computable if there is a programmable function f such that for every positive rational number r $|x - f/r/| < r$ holds .

DEF.7.:/39/. A real number x is called recursive computable if there are recursive functions a_n and b_n such that $1 < b_n$ and $0 \leq a_n < b_n$ for every n and $x = \sum_{n=0}^{\infty} a_n / c_n$ where $c_n = b_1 b_2 \ldots b_n$. /This sum is called generalized Cantor-sum./

DEF.8.:/55,60,65,77/. A real number x is recursive computable if there are recursive functions f,g,h and k such that $\forall n \forall m \left(m > k/n/ \Rightarrow |x_m - x| < 1/n \right)$ where $x_m = \frac{f/m/ - g/m/}{1 + h/m/}$

<u>DEF.9.:</u> The same as def.8, but $x_m = \pi(f/m/)$ where π is a fixed recursive bijection mapping from N to Q .

<u>DEF.10.:</u> The same as def.8. with $|r_n - r_{n'}| < 1/m$ for every $n, n' > k/m/$ .The last assertion can be changed to $\forall n, n' \; |r_n - r_{n'}| < 1/\min(n, n')$ or to $\forall n, n' \; |r_n - r_{n'}| < \frac{1}{n} + \frac{1}{n'}$.

<u>DEF.11.:</u> A real number x is recursive computable if the relations "$m/n > x$" and "$m/n < x$" are recursive .

<u>DEF.12.:</u> A real number x is recursive computable if there are recursive functions $f_1, f_2, g_1, g_2, h_1, h_2$ such that $x = \bigcap_{n=0} [r_n, s_n]$ where $r_n = \left(f_1/n/ - g_1/n/\right) / \left(1 + h_1/n/\right)$ and $s_n = \left(f_2/n/ - g_2/n/\right) / \left(1 + h_2/n/\right)$.

<u>DEF.13.</u> A programmable function f is a recursive process if $\forall r, s \in Q$, $s > 0$ and $r > 0$ then $|f/r/ - f/s/| < r + s$. Two recursive processes are equivalent if $|f_1/r/ - f_2/r/| < r$ for every positive rational number r. The equivalence classes of this relation are called recursive real numbers./cf./1/./

<u>THEOREM 1.:</u> a./ The above definitions are equivalent in case you allow total and partial recursive functions.

b;/ Denote by R_i the set of computable recursive real numbers in the sense of definition No i in case primitive recursive functions are allowed only . Then

$$R_2 = R_3 = R_{11} \subsetneq R_1 \subsetneq R_7 \subsetneq R_4 = R_6 = R_8 = R_9 = R_{10} \subsetneq R_{12} .$$

§.2.COMPUTABLE SEQUENCES

<u>DEF.14.:</u> /56/. A sequence x_k is recursive computable if there is a recursive function f such that $|x_k - f(k,n)/n| < 1/n$ for every k and n .

<u>DEF.15.:</u> A sequence x_k is recursive computable if there are a natural number p and a recursive function f such that $x_k = \sum_{n=0}^{\infty} f(n,k)/p^n$ and $\forall n,k: 0 \le f(n,k) < p$.

<u>DEF.16.:</u> A sequence x_k is recursive computable if there is a

recursive function f such that for every natural number $p > 1$ $\forall n \; 0 \le f/n,k,p/ < p$ and $x_k = \sum_{n=0}^{\infty} f/n,k,p/ p^{-n}$.

<u>DEF.17.:</u> A sequence x_k is recursive computable if the relation "$m/n > x_k$" is recursive .

<u>DEF.18.:</u> A sequence x_k is recursive computable if the relation "$m/n < x_k$" is recursive.

<u>DEF.19.:</u> A sequence x_k is recursive computable if there are recursive functions f,g,k and h such that

$$\forall n,m \left(i > k/n,m/ \Rightarrow \left| x_n - \frac{f/n,i/ - g/n,i/}{1 + h/n,i/} \right| < 1/m \right)$$

<u>DEF.20.:</u> /1/. A sequence x_n is recursive computable if there is a computable function [in the sense of def. 23] such that $x_n = f/n/$.

<u>THEOREM 2.:</u> Denote by S_i the set of computable sequences in the sense of the above i-th definitions [where arbitrary recursive functions are allowed]. Then $S_{19} = S_{20} = S_{14} \supsetneq S_{15} \supsetneq S_{16} \supsetneq S_{17} \cup S_{18}$.

§.3.COMPUTABLE FUNCTIONS

<u>DEF.21.:</u> /57/. A function $f: R \to R$ is recursive computable if there is a programmable function g such that $|f/r/ - g/m,r/| < \frac{1}{m}$ for every rational number r and natural number m.

<u>DEF.22.:</u> /65/. A function is recursive computable if Dom/f/ is an interval with computable recursive real endpoints and there are programmable functions g and h such that $\forall x,r,n \left(|x - r| < 1/h(n) \Rightarrow |f/x/ - g/n,r/| < 1/n . \right)$

<u>DEF.23.:</u> /1,10,11/. A function f is recursive computable if there is a recursive function F with the following properties:/i/ $f/x/ = y \Leftrightarrow F/N_x/ = N_y$ for every computable recursive real number x and y where N_x denotes the Gödel-number of x .

/ii/ if x is a computable recursive real number then so is $F/N_x/$.

/iii/ if N and M are Gödel numbers of x then $F/N/ = F/M/$.

DEF.24.: A function f is recursive computable if there are programmable functions h and g such that for every $r \in Q$
$\forall n,m \left(n > h/m/ \Rightarrow |f/r/ - g/n,\pi(n)/| < 1/m \right)$ ./cf.9,21/.

DEF.25.: A function f is computable /in recursive way/ if there are recursive functions g,k,h,u,v such that for every rational number x=p/q , y=r/s and natural number ℓ,m,n :
$|x-y| < 1/u(\ell) \Rightarrow |x_n - y_n| < 1/\ell$ and $n,m > v(\ell) \Rightarrow |x_n - x_m| < 1/\ell$
(where $x_n = (g/p,q,n/ - k/p,q,n/) / (1+h/p,q,n/)$
and $y_n = (g/r,s,n/ - k/r,s,n/) / (1+h/r,s,n/)$),
and $f/x/ = \lim x_n$. /cf./77/./

DEF.26.: The recursive computable functions are the equivalence classes of the equivalence relation $f \sim g \Leftrightarrow |f/m,r/ - g/m,r/| < 1/m$ (for every m,r) defined on the set of programmable functions./cf./14,21/./

DEF.27.: A function f is recursive computable if $f/x_k/$ is a computable sequence [in the sense of def.14.] for every computable sequence x_k. [In this case f is often called rec. comp. in the sense of Banach-Mazur].

DEF.28.: /27,65/. A function f is recursive computable if it is rec.comp. in the sense of the previous def. and there is a recursive function d : $\forall n, \forall x,y \in Dom/f/ : |x-y| < 1/d(n)$ implies $|f/x/ - f/y/| < 1/n$ and Dom/f/ is an interval with computable endpoints.

DEF.29.: In this definition I define the notion of recursive functionals /cf.25./ A functional θ maps from N^N to N^N. Basic functionals are the following functionals: 1. u/f/=f, 2. -/f,g/=f-g, 3. T/f,g/=f^g, 4. S/f/(x) = x+1 . We can build new functionals with the help of the following operations:

composition /=superposition/ : we get χ from θ and Φ :
$$\chi\langle f, g_1, \dots g_t\rangle (x_1, \dots x_{\ell-1}, x_{\ell+1}, \dots x_k, y_1, \dots, y_h) =$$
$$= \Phi\langle f\rangle (x_1, \dots x_{\ell-1}, \theta\langle g_1, \dots g_t\rangle (y_1, \dots y_h), x_{\ell+1}, \dots x_k)$$
repeating and using fictive variables, and the μ operator:
$$(\mu\phi)\langle f_1, \dots, f_k\rangle (y_1, \dots y_h) = \min\{z \mid \phi\langle f_1, \dots, f_k\rangle (z, y_1, \dots, y_h) = 0\}$$
A functional is recursive/or computable/ if it can be built from the basic functionals in finite steps.

DEF.30.: /25,61,65,75/. A function f is recursive cumputable if there is a recursive functional Φ such that $\forall a \in R \, \forall g \in N^N$
if $\forall k \in N \; |g(k)/k - a| < 1/k$ then $\forall k \in N \left| \frac{\Phi\langle g\rangle(k)}{k} - f/a/ \right| < \frac{1}{k}$

DEF.31.: /27/. A function f is recursive computable if for every recursive functions g,h, there are recursive functions g', h', ℓ' such that if $r_n = \frac{g/n/ - h/n/}{1 + \ell/n/}$ then $f/r_n/ = \frac{g'/n/ - h'/n/}{1 + \ell'/n/}$; and there is a recursive function d such that if $|x-y| < 1/d(n)$ then $|f/x/ - f/y/| < 1/n$ for every $x,y \in Dom/f/$ and natural number n.

DEF.32.: A function f is recursive computable if
/i/ f is continuous /ii/ $f/r_n/ = r'_n$ [see def.31.]
/iii/ there is a recursive function g such that
$\forall m,n,k \in N \, \forall a,b \in R$ if $r_n < a, b < r_m$ and $|a-b| < 1/g(m,n,k)$ then $|f/a/ - f/b/| < 1/k$ *

DEF.33.: A function f is recursive computable if
/i/ f is continuous /ii/ $\exists$ rec.funct. g : $\forall n,k \left| \frac{g/n,k/}{k} - f/r_n/ \right| < \frac{1}{k}$
/iii/ there is a recursive function g' such that $r_n < r_\ell, r_t < r_m$ and $|r_\ell - r_t| < 1/g'(m,n,k)$ implies $|g/\ell,k/ - g/t,k/| < 3$.

DEF.34.: A function f is recursive computable if
/i/ and /iii/ as in the previous definition
/ii/' $f/r_n/ = \lim_k \frac{g/n,k/}{k}$
AND:
/ii/'' the sequence is convergent is computable sense

DEF.35.: Let s_n a recursive listing of the intervals with rational endpoints. A function is recursive computable if
there is a recursive function g such that
/i/ $\forall a \in R \, \forall n \in N$ $a \in s_n \Rightarrow f/a/ \in s_{g/n/}$
/ii/ -"- $a \in s_n \Rightarrow \exists m \in N \; a \in s_m$ and $s_{g/m/} \subset s_n$
/iii/ $\forall n,k \in N$ $s_n \subset s_k$ and $n \geq k \Rightarrow s_{g/n/} \subset s_{g/k/}$

DEF.36.: A function f is recursive computable if there is a recursive function g such that
/i/ $\forall a \in R \, \forall m \in N$ $a \in s_m \Rightarrow f/a/ \in s_{g/n/}$
/ii/ $\forall a,b \in R$ $b \neq f/a/ \Rightarrow \exists n \in N \; a \in s_n$ and $b \notin s_{g/n/}$

DEF.37.: A function f is recursive computable if there are

* in these points $\{r_n \mid n \in N\} = Q$ a recursive listing

recursive functions d,s,a,b and g such that
if $g/N,M/ \leq n$ and $|x| < N$ then $|f/x/ - P_{N,M}/x/| < 1/M$
where $P_{N,n}/x/ = \sum_{j=0}^{d[N,n]} [-1]^{s[N,n,j]} a(N,n,j)/b(N,n,j) x^j$

THEOREM 3.: Denote by F_i the set of computable functions in sense of the definition 3.i. Then

$$F_{22}=F_{23}=F_{25}=F_{28}=F_{30}=F_{31}=\ldots=F_{37} \subsetneqq F_{21}=F_{24} \subsetneqq F_{26}$$
$$F_{28} \cap\!\!\!\!\!\neq F_{27}$$

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