

A csoport

1.

2.  $\sqrt{x^2 - y}$ , Gradient is  $\frac{x}{\sqrt{x^2 - y}}$   
 $-\frac{1}{2\sqrt{x^2 - y}}$

3.  $\arctan(x - 2y)$ , Gradient is  $\frac{1}{(x-2y)^2 + 1}$ ,  
 $-\frac{2}{(x-2y)^2 + 1}$

Hessian is  $-\frac{1}{((x-2y)^2 + 1)^2} (2x - 4y)$   $\frac{1}{((x-2y)^2 + 1)^2} (4x - 8y)$   
 $\frac{2}{((x-2y)^2 + 1)^2} (2x - 4y)$   $-\frac{2}{((x-2y)^2 + 1)^2} (4x - 8y)$

4.  $f(x, y) = y \ln(x^2 + 1) - y \ln(y) + y + \frac{2}{3}x^3$ ,

Gradient is  $2x^2 + 2x\frac{y}{x^2 + 1}$   
 $\ln(x^2 + 1) - \ln y$

$$2x^2 + 2x\frac{y}{x^2 + 1} = 0$$

$$(x^2 + 1) - y = 0$$

Solution is:  $[x = 0, y = 1], [x = -1, y = 2]$ ,

Hessian is  $4x + 2\frac{y}{x^2 + 1} - 4x^2\frac{y}{(x^2 + 1)^2}$   $2\frac{x}{x^2 + 1}$ , determinant:  $-\frac{1}{yx^2 + y} (4x^3 + 4x + 2y)$   
 $2\frac{x}{x^2 + 1}$   $-\frac{1}{y}$

$$\Delta(x, y) = -\frac{1}{yx^2 + y} (4x^3 + 4x + 2y)$$

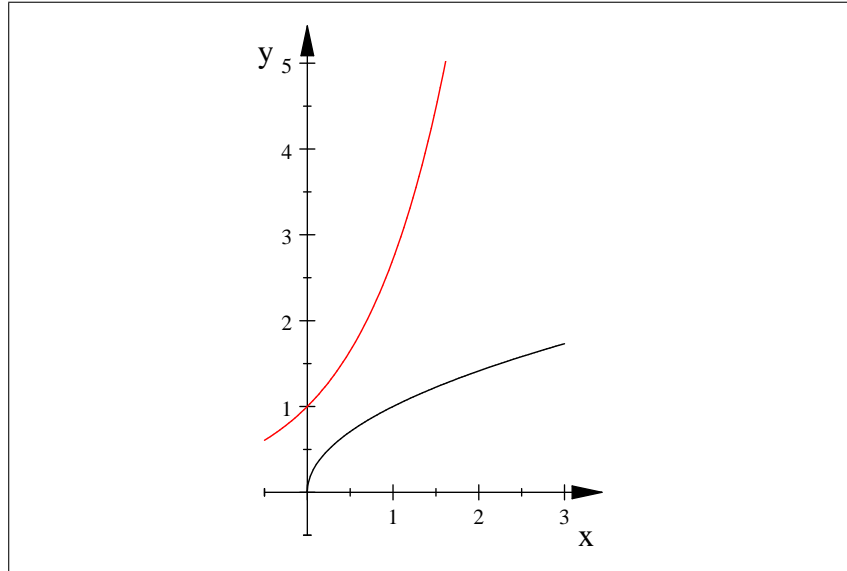
$$\Delta(0, 1) = -2$$

$$\Delta(-1, 2) = 1$$

$$D_1 f(x, y) = 4x + 2\frac{y}{x^2 + 1} - 4x^2\frac{y}{(x^2 + 1)^2}$$

$$D_1 f(-1, 2) = -4$$

5. ábra:

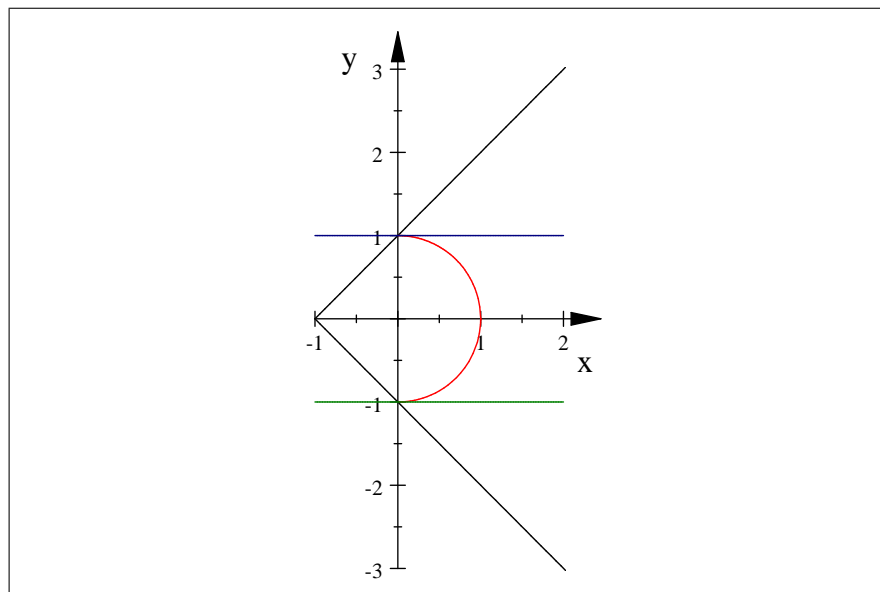


$$\int_0^1 \left( \int_{\sqrt{x}}^{e^x} 2xy dy \right) dx = \frac{1}{4}e^2 - \frac{1}{12}$$

$$\int_{\sqrt{x}}^{e^x} 2xy dy = [xy^2]_{\sqrt{x}}^{e^x} = xe^{2x} - x^2$$

$$\int_0^1 (xe^{2x} - x^2) dx = \frac{1}{4}e^2 - \frac{1}{12}$$

6.  $y = 1$ ,  $y = -1$ ,  $x = \sqrt{1 - y^2}$ ,  $x = |y| - 1$



B csoport

1.

$$2. \arctan(x - y^2), \text{ Gradient is } \frac{1}{(x - y^2)^2 + 1} - 2 \frac{y}{(x - y^2)^2 + 1}$$

$$3. \sqrt{2x - y}, \text{ Gradient is } \frac{1}{\sqrt{2x - y}} - \frac{1}{2\sqrt{2x - y}}$$

$$\text{Hessian is } \begin{pmatrix} -\frac{1}{(2x - y)^{\frac{3}{2}}} & \frac{1}{2(2x - y)^{\frac{3}{2}}} \\ \frac{1}{2(2x - y)^{\frac{3}{2}}} & -\frac{1}{4(2x - y)^{\frac{3}{2}}} \end{pmatrix}$$

$$4. f(x, y) = x \ln(y^2 + 1) - x \ln(x) + x + \frac{2}{5}y^5, \text{ Gradient is } \begin{pmatrix} \ln(y^2 + 1) - \ln x \\ 2y^4 + 2x \frac{y}{y^2 + 1} \end{pmatrix}$$

$$\begin{aligned} y^2 + 1 &= x \\ 2y^4 + 2x \frac{y}{y^2 + 1} &= 0 \end{aligned}$$

, Solution is:  $[x = 2, y = -1], [x = 1, y = 0]$

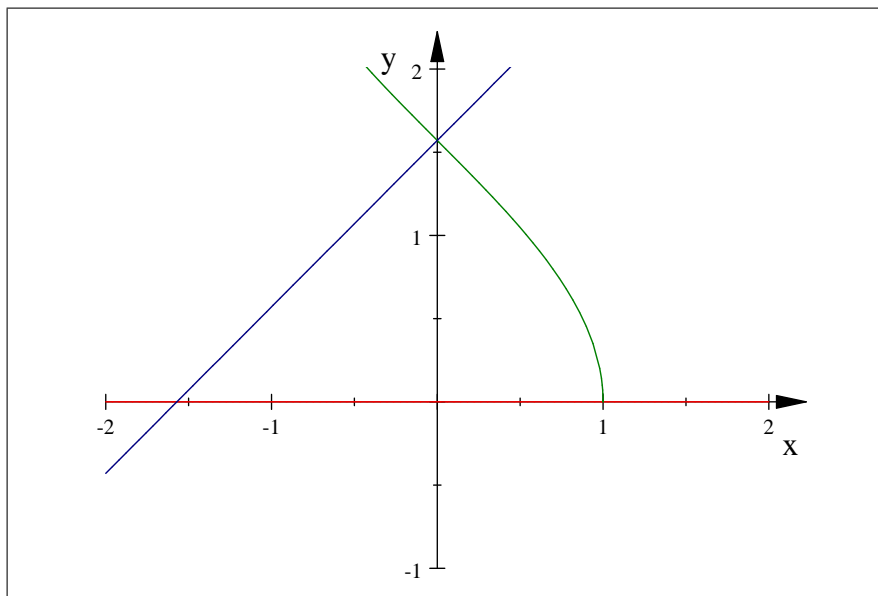
$$\text{Hessian is } \begin{pmatrix} -\frac{1}{x} & 2\frac{y}{y^2 + 1} \\ 2\frac{y}{y^2 + 1} & 2\frac{x}{y^2 + 1} + 8y^3 - 4x\frac{y^2}{(y^2 + 1)^2} \end{pmatrix}, \text{ determinant: } -\frac{1}{xy^2 + x} (8y^5 + 8y^3 + 2x)$$

$$\Delta(x, y) = -\frac{1}{xy^2 + x} (8y^5 + 8y^3 + 2x)$$

$$\Delta(2, -1) = 3$$

$$\Delta(1, 0) = -2$$

5. ábra:



$$\int_0^{\frac{\pi}{2}} \left( \int_{y-\frac{\pi}{2}}^{\cos y} y dx \right) dy = \frac{1}{2}\pi + \frac{1}{48}\pi^3 - 1$$

$$\int_0^{\frac{\pi}{2}} y \left( \cos y - \left( y - \frac{\pi}{2} \right) \right) dy$$

6.  $2 - |x|$

