Global stability and bifurcations in a delayed discrete population model

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Dedicated to Professor István Győri on the occasion of his 65th birthday

Abstract. We consider a family of difference equations used in population dynamics. First we recall the most relevant results concerning the global stability of the positive equilibrium. Then, using the survival rate as a parameter, we investigate the changes in the dynamics when it ranges between zero (semelparous populations) and one.

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1. Introduction

The aim of this paper is the study of different aspects of the dynamics of the difference equation

$$x_{n+1} = \alpha x_n + (1 - \alpha)h(x_{n-k}),$$

where $\alpha \in [0, 1)$, $k \geq 1$ is an integer, and $h : [0, \infty) \to [0, \infty)$ is a continuous function.

The motivations for our interest in Eq. (1.1) are mainly two:

First, this equation was proposed by K. R. Allen in 1963 to model whale populations. Since it was popularized by Clark in 1976, it is often referred to as Clark’s delayed recruitment model (see also [4, 17, 34] and references therein). In this context, $x_n$ represents the number of adult (sexually mature) members of the population in the year $n$, $\alpha \in [0, 1)$ is the annual survival rate, and $f = (1 - \alpha)h$ is the recruitment function, which is in general a nonlinear function of the size of population of adults a