

Mathematical Foundations of Economy
written exam No. 2.
2017. december 06. - Group A

Seminar leader:..... Your name:.....

Seminar dates:..... Neptun code:.....

The solutions of the Prompt Problems should be written just after the problems on this sheet. You must solve at least two of them **properly**, without significant error, otherwise the **total score** of your work is zero! You find the other problems on the reverse side of this sheet, for solutions use your empty papers. You should give detailed explanations and calculations, you may refer to theorems, too. Otherwise you can not get the total score. You have 90 minutes total for working.

Prompt problems:

a.) Give the derivative of the following function:

$$f(x) = \sqrt[4]{x + \sin(x)} \cdot \ln(5^x + \operatorname{ctg}(x)) \quad (3 \text{ points})$$

b.) Evaluate the following indefinite integral:

$$\int \frac{1}{\sqrt[7]{2x+13}} dx \quad (3 \text{ points})$$

c.) Evaluate the following indefinite integral:

$$\int \frac{2x^3 + 1}{7x^4 + 14x + 5} dx \quad (3 \text{ points})$$

1. **Problem:** Give the formula for the tangent line which meets the point of the graph of the below function corresponding to the abscissa $x_0 = 1$:

$$f(x) = xe^{x^2}, \quad x \in \mathbb{R} \quad (7 \text{ points})$$

2. **Problem:** Find the *global* extremal positions and calculate these extremal values of the function

$$f(x) = \log_2(x^2 + 1), \quad x \in [-1; 1] \quad (7 \text{ points})$$

3. **Problem:** Determine the intervals where the following function is **convex**:

$$f(x) = x^4 + 24x^2 - 11, \quad x \geq 0 \quad (7 \text{ points})$$

4. **Problem:** Evaluate the following limit:

$$\lim_{x \rightarrow 0} \frac{\tan(x)}{e^{2x} - 1} \quad (7 \text{ points})$$

5. **Problem:** Evaluate the following indefinite integrals:

(a)

$$\int \left(x^{2017} - 2\sqrt[3]{x} + \frac{10}{x} \right) dx \quad (6 \text{ points})$$

(b)

$$\int x^5 \ln(x) dx \quad (7 \text{ points})$$

Have many success!